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RESEARCH ARTICLE

Uncertainty Quantification in Load Forecasting for Smart Grids Using Non-Parametric Statistics

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ABSTRACT In flexibility markets, aggregators serve as crucial intermediaries by consolidating and selling consumer flexibility to grid operators or distribution system operators (DSOs). They are essential for grid management, offering load reductions based on power limits, and estimating expected consumer load in demand response scenarios. However, the inherent uncertainty in consumer behaviour poses a significant challenge, leading to deviations between projected and actual power consumption. In this context, this paper proposes a methodology for quantifying forecast uncertainties in power profiles at the aggregator level. The proposed methodology introduces a model-based approach to provide a more comprehensive representation of uncertainty and investigation of load variations. It provides load forecast values as comprehensive distributions, which are then sampled to generate newly sampled data from which the probability density function is extracted to quantify uncertainty, expressed by confidence intervals around the expected output. This approach aids in identifying the flexibility requirements for aggregated household power consumption, assists in quantifying uncertainties, and determines the flexibility needed for accurate forecasts of such consumption, which is essential for informed decision-making. The effectiveness of the proposed strategy is demonstrated using a synthetic dataset to assess its capability to quantify uncertainties in probabilistic forecasts. Additionally, a potential case study with a neighborhood of 14 houses connected to the same distribution transformer is presented to validate the proposed method. A comparative investigation of quantified uncertainties is presented by employing the Additive Gaussian Process (AGP), the *Prophet* forecasting, and the quantile regression, highlighting the usefulness of the proposed approach in flexibility markets. The results demonstrated the superiority of AGP-based load forecasts and flexibility needs with precise prediction accuracy. The comparative study demonstrates that the proposed method with AGP presents a minimum uncertainty when forecasting the total residential load than other benchmark models with a percentage of 26% and 21% in mean absolute error, respectively, for the different datasets. The continuous ranked probability score also revealed a 39% increase in the accuracy of probabilistic forecasts via the proposed method in contrast to others.

INDEX TERMS Additive Gaussian process, facebook *Prophet* model, flexibility markets, forecasting analysis, uncertainty analysis.

NOMENCLATURE

PARAMETERS

α Percentile.

$\ell_{EQ, \eta EQ}$ Hyperparameters of EQ.

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ℓ_M, η_M	Hyperparameters of M.
ν	Positive hyperparameter.
σ	Variance.

VARIABLES

β_τ	Regression coefficients specific to the quantile τ .
ϵ_t	Error Term.
$\mathcal{O}(N^3)$	Covariance matrix.
u	Residual.
X	Covariates.
x_c	Vector calendar variables.
x_w	Vector weather variables.
y	Aggregated power (kW).
y_{lim}	Power limit (kW).

FUNCTIONS

Γ	Gamma function.
ρ_τ	loss function.
F^{-1}	Inverse Cumulative Distribution Function.
$g(t)$	Trends of non-periodic changes.
k_c	Kernel calander fucntion.
k_w	Kernel weather function.
k_{so}	Second order kernel function.
$s(t)$	Nonlinear function on a daily, weekly or yearly.

ABBREVIATIONS

AGP	Additive Gaussian Process.
ANN	Artificial Neural Network.
CDF	Cumulative Distribution Function.
CI	Confidence Interval.
CRPS	Continuous Ranked Probability.
DSM	Demand Side Management.
DSO	Distribution System Operator.
EMS	Energy Management System.
EQ	Exponential Quadratic kernel.
KDE	Kernel Density Estimation.
MAE	Mean Absolute Error.
M	Matérn kernel.
PDF	Probability Density Function.
PICP	Prediction Interval Coverage Probability.
QR	Quantile Regression.
RMSE	Root Mean Squared Error.
sMAPE	squared Mean Absolute Percentage Error.
STLF	Short-Term Load Forecasting.
SVM	Support Vector Machines.
VarS	Variogram Scores.

I. INTRODUCTION**A. BACKGROUND AND MOTIVATION**

Uncertainty in forecasts in recent years has been an important aspect of many fields, including statistics, economics, weather prediction, and machine learning. It reflects the inherent unpredictability or variability in future outcomes, and it's crucial to understand and quantify this uncertainty

to make informed decisions [1]. However, uncertainty arises from various factors, including seasonal variations, weather conditions, economic fluctuations, customer behaviour, the model's parameters, and unforeseen events. Therefore, it is essential to develop forecasting models and methodologies that can handle these uncertainties and provide reliable forecasts [2]. On the other hand, electricity grids in cold regions face unique challenges compared to those in milder climates. Electric space heating significantly complicates the energy demand profile, especially in cold regions.

In these areas, the energy demand is further complicated by the heavy reliance on electric space heating and water heating. Occupant behaviours can also significantly impact electricity consumption patterns, and the unpredictability of these behaviours introduces uncertainty in load forecasting. For instance, daily routines throughout the day, changes in work schedules, sleep patterns, and other activities can vary, leading to fluctuations in electricity usage. Additionally, the increasing use of electric vehicles (EVs) adds to the fluctuating energy demands [3]. This creates a demanding landscape that experiences both seasonal and daily peaks. Cold winters contribute to significant seasonal spikes in power consumption, while daily patterns add further variability, putting a strain on local distribution networks [4], [5], [6]. In the context of local flexibility markets that enable the trading of resources that can provide flexibility to the electricity grid, it is crucial to tackle these challenges, particularly the uncertainties surrounding load [7]. These markets serve as vital channels for making real-time adjustments to electricity consumption, offering potential solutions to congestion management and peak shaving [8]. Making these real-time adjustments becomes imperative to maintain grid stability and ensure uninterrupted power supply during periods of high demand [9]. Within this ecosystem, the accuracy of the aggregated load estimation in a neighbourhood is paramount [10] for cold weather regions, as inaccuracies can lead to economic inefficiencies and potential grid challenges. If overestimated, it might fail to reduce consumption as promised, which can lead to penalties and grid instability. On the other hand, underestimating flexibility means missing out on market opportunities and potential revenue [11]. While traditional methods of load estimation have their merits, they increasingly fall short when confronted with the daily consumption intricacies in cold regions. The combined impact of electric space and water heating systems, the rising tide of EVs, and the nuanced changes in consumption caused by the vicissitudes of cold climates necessitate an approach to the prediction of aggregated load with uncertainties [12].

Short-Term Load Forecasting (STLF) serves as a pivotal tool in addressing load demand for optimal electricity market planning, as demonstrated by its extensive application in delivering usage plans [13]. Furthermore, its efficacy extends to the Energy Management System (EMS), playing a crucial role in real-time load consumption prediction for the implementation of effective Demand Side Management

(DSM) programs aimed at enhancing energy efficiency [14]. The primary goal of such initiatives is the reduction of end-user electricity consumption by strategically modifying load patterns, particularly during peak times [15]. However, upon a critical examination of the current state of the art, certain gaps and inadequacies come to the forefront, particularly when viewed through the lens of modern smart grids [16]. The existing body of literature predominantly explores STL using deterministic or probabilistic models [17]. Additionally, the surge in popularity of machine learning techniques, particularly neural networks and data-driven methods like Artificial Neural Networks (ANN) [18], Support Vector Machines (SVM) [19], and Gaussian Process (GP) [20], has been noteworthy in recent years for forecasting aggregated load. These techniques promise the ability to capture non-linear patterns but reveal a common weakness in uncertainty quantification when subjected to a critical evaluation.

Despite their proficiency in forecasting, these machine learning models often struggle to provide reliable uncertainty estimates. Their inherent “black-box” nature, combined with the risk of overfitting, introduces unpredictability into forecasts, particularly in the face of anomalous events or rapid grid changes. Furthermore, the reliance of supervised learning algorithms on training datasets with precise forecasts introduces uncertainties that limit their practical application. This leads us to a fundamental question: How can uncertainties be effectively incorporated into forecast models to enhance their reliability and applicability in dynamic energy environments?

B. LITERATURE REVIEW

Point or deterministic forecast methods have been widely used historically because of their simplicity and understandable employment [21]. However, these deterministic methods are gradually replaced by probabilistic methods that respond to the stochastic factors corresponding to the system’s flexibility [22]. The methods proposed by those works suffer from two issues: the first is the accumulation of errors due to the stochastic behaviour of end-users, and the second is the insufficiency of the model to provide reliable forecasts of users with different power patterns for an ensemble of houses since uncertainties can significantly impact the actual demand [23]. Various advanced probabilistic load forecasting methods have prominently emerged in recent years. While prior research has not explicitly addressed uncertainty propagation from systems, notable progress has been made in this forecasting domain. For instance, [17] investigates the propagation of input uncertainty, recognizing the challenges involved in predicting outputs. In this context, [16] examines the outputs of machine learning algorithms to quantify uncertainty in determining future power demand changes.

On the other hand, the application of Gaussian processes network-based models, as highlighted in [24], stands out for

its ability to generate empirical distributions by sampling multiple predictions. This method is effective, and analytical distributions prove valuable for gradient-based design, by minimizing the need for extensive predictions. Focusing on a forecast horizon of 24 hours, this approach estimates load confidence intervals based on quantiles derived from past forecast errors. This method’s adaptability extends to security analyses of power systems, demonstrating its capacity to generate demand scenarios at specified risk levels. The primary objective of this analysis is to understand system reactions to electricity use ramps and periods of low load [25], [26]. In practice, three strategies are often used to communicate uncertainty in load forecasts that allow a more comprehensive exploration of uncertainties in load predictions: scenario forecasting [32], interval forecasting [27], and quantile forecasting [2], [28].

Table 1 provides a comprehensive overview of how each reference navigates the complexities of uncertainty within the context of load forecasting. By examining the entries in the table, one can discern the diverse methodologies and approaches employed by different authors to address uncertainties. Reference [31] incorporates neural network models and applies confidence interval-based uncertainty quantification for electricity price forecasting. The authors combine different analyses for time series analyses (statistical) by applying uncertainty to clustered data for power to better detect trend shift (concept drift) and handle the noise in data more precisely. Another study [28] builds upon methodologies from competition winners, integrating quantile regression and neural networks for load and price forecasting. Authors in [7] emphasize the robustness of a model in handling missing data and outliers and adapting to trend changes. The authors thoroughly examine the model’s mechanisms for estimating uncertainty, providing confidence intervals, and evaluating reliability in scenarios where uncertainty plays a pivotal role [33].

While quantifying forecast uncertainty may support better decision-making in the energy industry, there have been few journal articles published on quantifying forecast uncertainty [34]. Uncertainty in load stems from various exogenous factors such as temperature, humidity, and solar radiation. It is also attributed to the temporal dynamics, encompassing seasonality, trends, and cyclic patterns [17]. Several works have been carried out [16], [29], [31] encompassing temporal dynamics or exogenous factors; however, a notable distinction is made regarding the application of uncertainty quantification that did not explicitly consider the practical implementation or utilization of their uncertainty quantification methods. The interplay of these factors introduces variations in the data quality and quantity incorporated into the forecasting models. As emphasized in the literature, the impact of exogenous factors on uncertainty analysis is significant, and understanding this relationship becomes paramount in refining forecasting methodologies [35]. Striking a balance between the richness of data and the potential influence of exogenous variables is essential, given that an

TABLE 1. The effective elements of uncertainty in forecasting procedures according to the relevant literature.

References	Modeling approach	Non parametric models	Uncertainty analysis	Temporal Dynamics	Exogenous factors	Application
[7]	Local flexibility market mechanism	✓	✓	✗	✓	DSOs flexibility services and quantify the financial benefits
[17]	Trajectory forecasting	✓	✓	✓	✗	Improving mean prediction accuracy
[16]	ML and ensemble learning	✓	✓	✓	✗	Future power demand changes
[21]	Deterministic	✗	✓	✗	✗	Manage user expectations
[22]	Statistical models	✓	✓	✓	✗	Wind power DM
[24]	Gaussian Process Regression	✓	✗	✗	✓	Reduce the peak energy demands and energy supply risks
[25]	Markov-chain mixture distribution (MCM)	✓	✓	✗	✗	react to ramps of electricity use
[26]	NN- based CIs	✓	✓	✗	✗	DM and risk management in energy systems
[27]	Neural Network	✓	✓	✓	✗	Required power reserve (PV)
[28]	Quantile regression neural network	✓	✓	✗	✓	✗
[29]	Five algorithms of ensemble learning	✓	✓	✓	✓	Produced good estimates of the confidence in a forecast
[30]	Bootstrap aggregating	✓	✗	✓	✗	Improving forecasting load
[31]	Combined unsupervised ensemble learning	✓	✓	✓	✓	Detect trend shift better and handle the noise in data more precisely
Our work	Additive Gaussian Process	✓	✓	✓	✓	Calculating flexibility needs

DM: Decision Making, ML: Machine Learning, CIs: confidence intervals, PV: Photovoltaic Power

increase in observed data may mitigate model noise, yet the inherent process noise remains linked to the underlying data-generating process, maintaining its level of uncertainty, especially when data points are scarce [36].

C. CONTRIBUTIONS AND ORGANIZATION

The main objective of our study is to predict load patterns while accurately representing and quantifying the uncertainties inherent in the power consumption forecasting process. As mentioned earlier, the inherent presence of uncertainties stemming from various sources necessitates a forecasting methodology with uncertainty quantification. This quantification is crucial for calculating flexibility requirements for aggregators in the energy markets. Accordingly, the contributions of this work are twofold:

Proposing a methodology for load forecasting that incorporates the estimation of uncertainties. This methodology not only generates forecasts but also provides measures of uncertainty associated with each confidence interval. By leveraging the probabilistic nature of the Additive Gaussian Process (AGP), our approach inherently quantifies the uncertainty in predictions, offering detailed insights into the reliability of the forecasts.

Forecasting load and their associated uncertainties are analyzed for each 15-minute forecast interval over a 24-hour period. By utilizing probabilistic models to generate posterior predictive samples, we gain a more precise understanding of the reliability of the load forecasts. This approach supports informed decision-making processes by transparently reporting conditional expectations and confidence intervals, which is essential for effectively conveying flexibility requirements.

This transparency aids in managing loads and reducing the Distribution System Operator's (DSO) network operational costs.

The proposed methodology utilizes the AGP for performing load forecasting, and uncertainty quantification [37], [38], [39]. To evaluate its efficacy, a comparative study is presented with a modular regression model, also known as the *Prophet* model [40] and quantile regression [28], [41]. The modeling accuracy is evaluated through several metrics for scoring the forecasting methods. Subsequently, the uncertainty quantification calculating the flexibility need is carried out utilizing the confidence interval width and inverse CDF. Comparative analysis is effectuated on two case studies: (i) on the synthetic dataset of 1000 houses located in Quebec and (ii) on a low voltage network consisting of 14 houses fed by the same transformer.

The rest of the paper is organized as follows: Section II presents the proposed methodology in detail. Section III formulates the forecasting models utilized in the methodology. Section IV presents results and discussions of the two case studies. An investigation of uncertainty quantification has been presented for two case studies, followed by the conclusion in Section V.

II. METHODOLOGY

The proposed methodology is aimed to address net demand uncertainty that requires a careful evaluation of various factors to ensure effectiveness and suitability. Also, it helps mitigate risks associated with net demand uncertainty and assesses its ability to provide actionable insights and support decision-making. The proposed methodology introduces

a model-based approach that offers load forecast values as comprehensive distributions, allowing a more nuanced understanding of uncertainty by capturing the variability and range of possible outcomes. Moreover, in the context of flexibility markets, where aggregators play a crucial role in managing load and providing flexibility services, understanding uncertainty is paramount. This methodology enables stakeholders to identify and quantify flexibility requirements based on probabilistic forecasts. This informs decision-making processes, allowing for better allocation of resources and mitigation strategies for potential deviations between projected and actual power consumption.

Figure 1 shows the methodology to tackle the task of forecasting household power consumption, focusing on quantifying uncertainties and calculating the flexibility needs. It is divided into four distinct phases. The first phase starts by gathering historical power consumption data and capturing diverse load patterns and trends during winter. The consumption data includes both flexible and non-flexible loads as well. A flexible load refers to the electricity consumption that can be adjusted or shifted in time without significant inconvenience or cost. This contributes mainly to the flexibility process and can be exploited by the aggregator to balance the grid, especially during peak demand periods. Examples of flexible loads include space and water heating, washing machines, and dryers. Non-flexible loads, on the other hand, are those that cannot be easily adjusted or shifted without causing significant disruption, for instance, essential lighting systems. In addition to consumption data, this process considers external factors such as weather conditions and calendar time. Note that the node locations are not considered as they increase the complexity of the models under investigation with larger datasets. Analysis of uncertainty was incorporated to capture these insights and to determine the solution to such a problem in order to have an accurate forecast for decision-making. Initial data analysis is facilitated through non-parametric statistical techniques based on a forecasting model to uncover underlying patterns and variances in the dataset. Here, the learning process involves training the model using historical data, where hyperparameters are estimated to optimize forecasting accuracy. The yield of the first phase is a tuned forecast model. In this work, AGP [37], [38] and *Prophet* [40] and quantile regression models are considered for load forecasting with uncertainties. The second phase consists of utilizing the tuned forecasting model predictions in the sampling process.

As AGP is a probabilistic model, this phase is crucial in understanding the mechanics of the forecasting model that captures the inherent uncertainty in the function estimation by including probabilistic components. Leveraging the power of Bayesian statistics, AGP provides a flexible framework for capturing uncertainties and relationships in the data. When utilized for prediction, AGP generates point estimates and also provides a complete posterior distribution for the predicted values from the priors and the likelihoods by

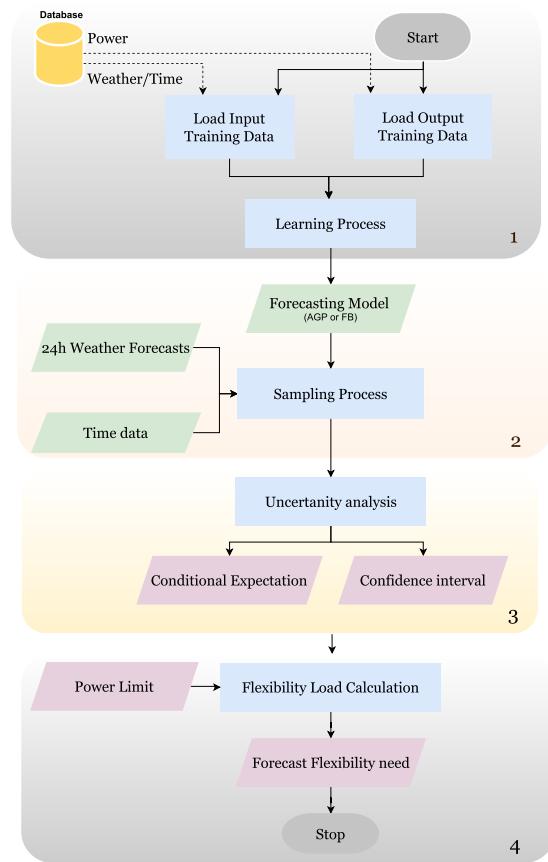


FIGURE 1. Flowchart of the proposed methodology.

incorporating the uncertainties. The posterior distribution encapsulates a range of plausible outcomes, reflecting the model's uncertainty about the true values of the predictions. By sampling from this distribution, we generate a multitude of potential scenarios, each respecting the uncertainty present in the model. These sampled data points, often referred to as posterior predictive samples, enable a more comprehensive understanding of the potential outcomes and aid in making informed decisions. This Bayesian approach not only provides a point forecast but also equips us with the tools to assess the range of possibilities and make robust decisions based on the inherent uncertainties in the data. Note that for the *Prophet* there are three sources of uncertainty in the forecast: uncertainty in the trend, uncertainty in the seasonality estimates, and additional observation noise. The uncertainty in the seasonality estimates can be extracted by Bayesian sampling to get the posterior predictive sampled data.

In the third phase of analyzing the uncertainty, the sampled data (posterior predictive samples), with its components of conditional expectation and confidence intervals, plays a pivotal role in conducting uncertainty analysis within a Bayesian modeling framework. The conditional expectation serves as a central tendency measure, representing the

average prediction for new, unseen data points. However, acknowledging the inherent uncertainties in real-world data, confidence intervals derived from the posterior predictive distribution become invaluable. These intervals provide a quantifiable range of plausible values for predictions, effectively expressing the uncertainty associated with the model. They encapsulate the spectrum of potential outcomes and offer decision-makers insights into the variability inherent in the predictions. Consequently, in this work, this comprehensive uncertainty analysis, incorporating both conditional expectation and confidence intervals, empowers users to make informed decisions on the possible power profiles, considering the full spectrum of possibilities and acknowledging the uncertainties inherent in the underlying data and modeling assumptions.

The fourth phase utilizes a critical maximum consumption threshold (power limit). This threshold, influenced by grid capacity, consumer demand patterns, and environmental factors, is essential for evaluating flexibility in energy markets. By quantifying the difference between forecasted power consumption and this maximum threshold, we assess the required flexibility. This investigation enhances flexibility load calculation, addressing potential reductions in power consumption proactively. This calculation, relying on conditional expectation and confidence intervals, reveals the flexibility needed for the ensemble of houses by taking into account the inherent uncertainty. These insights guide further actions by the aggregator in participating effectively in the flexibility market, ensuring power demand stays within manageable bounds.

III. FORECASTING MODELS

A. ADDITIVE GAUSSIAN PROCESS FORECASTING MODEL

AGPs are a class of models that have gained popularity in machine learning and statistics. Realizations from an AGP correspond to random functions, and consequently, AGPs naturally provide a prior for an unknown regression function that is to be estimated from data. By definition, the prior probability density of AGP function values $f(X) = (f(x_1), f(x_2), \dots, f(x_N))^T$ for any finite number of fixed input covariates $X = (x_1, x_2, \dots, x_N)$ where $x_i \in \mathcal{X}$ is defined to have a joint multivariate Gaussian distribution [42]:

$$f(x) \sim \mathcal{N}(0, K_{X,X}(\theta)) \quad (1)$$

The elements of the N -by- N covariance matrix are determined by the AGP kernel function, denoted as $[K_{X,X}(\theta)]_{i,j} = k(x_i, x_j | \theta)$, where θ represents the parameters. In general, the mean in 1 can depend on X , but in practice, a zero mean is often assumed. The covariance, also known as the kernel function, of the normal distribution governs the smoothness of the function f , indicating how rapidly the regression function can change. While AGP is formulated such that any finite-dimensional marginal follows a Gaussian distribution, AGP regression is considered a non-parametric method since the regression function f lacks an explicit

parametric form [43]. More precisely, AGP encompasses a countably infinite number of parameters that define the regression function, corresponding to the function values f at all possible inputs.

In this case study, the forecast is based on the AGP approach, where the kernel (covariance) is expressed as a sum of kernels. In this additive structure, each kernel models the effect of individual covariates or their interactions. Intuitively, each AGP component f now represents a nonlinear function that characterizes the corresponding effect, and the cumulative impact of multiple covariates is the sum of these nonlinear functions. This is achieved by employing specific kernels tailored to different types of covariates. Subsequently, the AGP model, resulting from this configuration, can be explained by,

$$k(x) = k_w(x_w) + k_c(x_c) + k_{so}(x), \quad (2)$$

where k_w and k_c describe the weather and calendar-related kernels, respectively, k_{so} stands for the second-order kernel compounds. The two first elements contain all first-order composites about temperature, humidity, solar radiation, time of day, and day of the week, based on the input dimensions. In this regard, the calendar and weather variables are included in the vectors, respectively. In (2), the vectors x_w and x_c contain weather and calendar variables, respectively. More details can be found in [37] and [38]. The hyperparameters of the Exponential Quadratic kernel (EQ) continuous covariates are dedicated to the weather-related component. The weather kernel is depicted in (3),

$$k_{EQ}(x, x') = \eta_{EQ}^2 \exp\left(-\frac{(|x - x'|)^2}{\ell_{EQ}^2}\right). \quad (3)$$

However, Matérn 5/2 (M) functions would be the kernel for the calendar component as shown in (4),

$$k_M(x - x') = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu} \frac{x - x'}{\ell_M}\right)^\nu K_\nu \left(\sqrt{2\nu} \frac{x - x'}{\ell_M}\right). \quad (4)$$

In this kernel, ℓ and ν are both positive hyperparameters. The Half Cauchy is utilized as the prior over the variance σ . Gamma ℓ_{EQ} and ChiSquared ℓ_M distributions are used as the prior over the hyper-parameters of the EQ and Matérn functions, respectively. Particularly, they are used to explain the lengthscales, η_{EQ} and η_M , of these covariance bases. Additionally, The third symbolizes a compositional first-order kernel that is intended for searching interactions between multi-dimensional variables [37].

Additionally, the goal of Bayesian inference is to compute the posterior distribution over the function $f(x)$ evaluated at arbitrary test inputs x . For Gaussian likelihoods, the posterior distribution takes a convenient closed-form solution, thus the predictive distribution at a test variable. However, it is difficult to compute in practice when N is large. The computational cost of matrix inversion is in $\mathcal{O}(N^3)$. Naively, these operations each incur $\mathcal{O}(N^3)$ computations, as well as $\mathcal{O}(N^2)$

storage for each entry of the kernel (covariance) matrix, often starting with a Cholesky decomposition. To resolve those issues in the configuration-based AGP a scalability analysis has been applied. It should be noted that the choice of the values of hyperparameters is the same as in our previous work [37], which is random search method. While in straightforward conditions, the Bayesian approach might be unproductive, most applications of AGPs rely on engineering sophisticated hand-crafted kernels involving many hyperparameters where the risk of overfitting is pronounced. A more robust solution is to incorporate confidence intervals that reflect these uncertainties in the model choice. Initially, we have a prior distribution that predicts the aggregated power. As the data is gathered, we refine this to include only functions that align with the observations, creating a posterior distribution. This posterior is essentially an updated prior, incorporating new data. Each new piece of data further improves this process. The AGP, in this context, describes a probability distribution across a range of potential functions that match a given set of points. This model allows us to determine mean values for these functions and assess the confidence of these predictions through variance. The function (posterior) is continuously updated with new data. The AGP represents then a probability distribution across possible functions, where any subset of these functions follows a joint Gaussian distribution. Meanwhile, for the regression predictions, the mean function derived from the posterior distribution is used. More in-depth details of AGPs are available in [37] and [44].

B. PROPHET FORECASTING MODEL

A modular regression model popularly known as the *Prophet* model was developed by Facebook [40]. It is built to handle time-series data with varied seasons [45], [46], and offers a versatile framework for deriving confidence intervals to determine the uncertainty inherent in the prediction system. Specifically, it is based on an additive model composed of three components: the trends $g(t)$ simulating non-periodic changes in the data, the seasonality $s(t)$ describing nonlinear behaviour on a daily, weekly, or yearly basis, and the third is the error term ε_t represents the distinctive features of the data improving the accuracy. *Prophet* is significant for forecasting comparison as it is well-known for time-series prediction. Power forecasting often exhibits trends and seasonal patterns. *Prophet* explicitly models these trends and seasonality (daily, weekly, yearly) and can incorporate holidays and events, making it a good contrasting comparative model. Mathematically, in this study, the decomposed time-series model comprising two fundamental components is utilized to scrutinize power consumption patterns across a group of households [47]:

$$y(t) = g(t) + s(t) + \varepsilon_t. \quad (5)$$

Equation (5) doesn't use traditional logistic regression for its growth modeling, but it employs an adaptive approach to effectively capture the growth patterns in the data. The trend

function $g(t)$ is a nonlinear saturating function modeled using the logistic growth function, given by:

$$g(t) = \frac{c(t)}{1 + e^{-k(t-m)}}, \quad (6)$$

where $c(t)$ is a time-varying consumption per day, k denotes a varying growth rate and m is the offset parameter. The periodic effect of yearly seasonal variations is modeled using the Fourier series; hence, an approximate smooth seasonal effect is tied with a standard Fourier series represented as:

$$s(t) = \sum_{n=1}^N \left(a_n \cos \frac{2\pi nt}{p} + b_n \sin \frac{2\pi nt}{p} \right), \quad (7)$$

where p is the period of the seasonality, it can be 365.25 or 7 for yearly and weekly seasonality, respectively. The Prophet model is designed to auto-tune the hyperparameters with grid search/cross-validation, and the training splits the data in two: (i) timestamps containing the time and date details, (ii) the logged values.

C. QUANTILE REGRESSION FORECASTING MODEL

Quantile regression (QR) offers distinct advantages over ordinary regression models, particularly when dealing with data that exhibit high variability in response measurements [48]. QR estimates conditional quantiles, making it more robust in scenarios where the data are heteroscedastic or not normally distributed.

In the context of time series power data analyzed in this paper, QR involves modeling the relationship between predictor variables (matrix x) and the dependent variable (vector y) across different quantiles of interest. The input variables representing the predictors include weather-related components, namely temperature, humidity, and solar radiation, as well as calendar-related variables, such as the hour of the day and the day of the week. To characterize the periodicity, the time index is mapped onto a two-dimensional input through $\begin{pmatrix} \cos(g(t)) \\ \sin(g(t)) \end{pmatrix}$. The periodicity is controlled by the function $g(t) = \frac{2\pi t}{\tau}$, where τ is a period. Normally, τ is fixed to match weekly and daily patterns. Two different periods are used in this work $\tau = 24$ and $\tau = 24 \times 7$ to generate 4 calendar variables as proposed in [6].

The QR model [49], generalizes the linear regression framework by allowing the estimation of conditional quantiles. Specifically, for a given quantile τ of the dependent variable y , the linear QR model is expressed as:

$$Q_\tau(y|x) = x\beta_\tau, \quad (8)$$

where $Q_\tau(y|x)$ denotes the τ -th quantile of y , x is the matrix of predictor variables, and β_τ represents the regression coefficients specific to the quantile τ .

To estimate β_τ , QR solves the following optimization problem using an asymmetric loss function ρ_τ :

$$\hat{\beta}_\tau = \arg \min_{\beta} \sum_{i=1}^n \rho_\tau(y_i - x_i^T \beta), \quad (9)$$

where y_i and x_i are the i -th observations of y and x , respectively, and ρ_τ is defined as:

$$\rho_\tau(u) = u(\tau - I(u < 0)). \quad (10)$$

This loss function ρ_τ penalizes deviations differently depending on whether u (the residual) is negative or positive relative to τ . In summary, QR provides a flexible framework for analyzing the relationship between predictors and a response variable across different quantiles, thereby capturing the variability and distributional characteristics of the data more effectively than traditional regression methods.

IV. RESULTS AND DISCUSSION

A. CASE STUDY - 1000 HOUSES

1) DATA AND ANALYSIS SETUP

In this work, simulations are conducted using load data sourced from aggregate simulated end-user profiles of 1000 residential houses. The database with a sampling interval of 15 minutes is administered by Hydro-Québec, a research institution situated in Québec. The specified time covers the period from December 1, 2018, to December 31, 2019. Additionally, this study incorporates temperature, humidity, and solar radiation data from the same geographic location within demand areas. As illustrated in Figure 2,

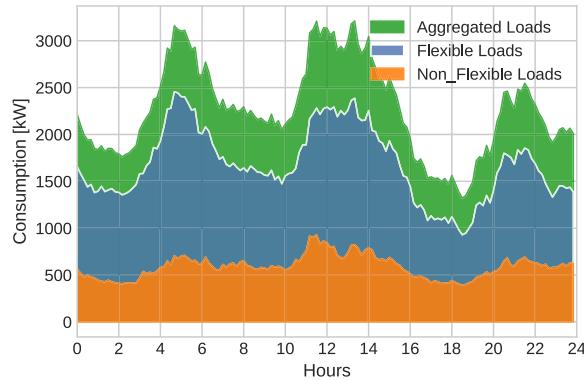


FIGURE 2. Power consumption by flexible and non-flexible loads within the total load of the 1000 houses for a specific day.

it is evident that flexible loads, namely air conditioners and heating systems, constitute the major part of the rated building load. The remaining loads are assumed to be non-flexible to facilitate Demand Response (DR). Figure 2 presents the cumulative stacked graph with a peak load of 3000 kW observed at 5 AM, which encompasses the total electrical demand, including both flexible and non-flexible loads. To analyze variations in these load profiles, statistical methods were applied to the dataset. Initially, the focus of the forecasting and uncertainty estimation was on non-flexible loads, a process that introduced a certain level of additional uncertainty into the results. Later, the analysis was expanded to include the entire aggregated load, thereby covering both flexible and non-flexible load types.

2) FORECASTING PERFORMANCE

Forecasting load demand depends on factors like the number of households and various infrastructure components. However, consumption related to non-flexible loads, namely lighting, major household appliances, and electronics, for a horizon of 24 hours, follows a highly stochastic pattern. The overall demand in a neighborhood can be quite uncertain, primarily due to the presence of significant non-flexible loads, posing significant challenges for grid management and the behaviors of occupants that change depending on the calendar variables. Consequently, error and uncertainty are interconnected yet separate facets in measurement characterization. An error signifies the variance between a measurement outcome and the actual value of the power. In contrast, uncertainty gauges the confidence in the assertion that the power forecasting result accurately reflects the power value, encompassing various factors influencing reliability. These terms jointly define the precision of measurements. Hence, this work performs load forecasting with uncertainty through AGP and the Prophet model for aggregated power consumption profiles encompassing flexible and non-flexible loads and aggregated non-flexible loads for an ensemble of 1000 houses.

Accuracy's metrics: The efficiency of the two employed models is evaluated using a variety of statistical parameters. Table 3 presents the mean of the metrics of all the generated predicted profiles, including mean absolute error (MAE) (11), root mean square error (RMSE) (12), coefficient of determination (R^2) (13), and squared mean absolute percentage error (sMAPE) (14).

$$MAE = \frac{1}{N} \sum_{t=1}^N |y_t - \hat{y}_t|, \quad (11)$$

$$RMSE = \left\{ \frac{1}{N} \sum_{t=1}^N (\hat{y}_t - y_t)^2 \right\}^{1/2}, \quad (12)$$

$$R^2 = 1 - \frac{\sum_{t=1}^N (y_t - \hat{y}_t)^2}{\sum_{t=1}^N (y_t - \bar{y})^2}, \quad (13)$$

$$sMAPE = \frac{100\%}{N} \sum_{t=1}^N \frac{|\hat{y}_t - y_t|}{(|\hat{y}_t| + |y_t|)/2}, \quad (14)$$

where \hat{y} and y present predicted and actual power usages for N discrete-time samples. Moreover, to evaluate the accuracy of probabilistic forecasts, Prediction Interval Coverage Probability (PICP), Continuous Ranked Probability (CRSP) [50], and Variogram scores [51]. PICP metric evaluates the coverage of prediction intervals by measuring the proportion of observed outcomes falling within the forecasted prediction intervals.

$$PICP = \frac{1}{N} \sum_{i=1}^N 1(y_i \in [L_i, U_i]), \quad (15)$$

where N is the total number of observations. y_i is the actual value for observation. L_i and U_i are the lower and upper

bounds of the prediction interval for observation. 1 is the indicator function, which returns 1 if the condition inside is true and 0 otherwise. CRPS measures the discrepancy between the Cumulative Distribution Function (CDF) of the forecast distribution and the CDF of the observed outcomes. It quantifies the overall accuracy of probabilistic forecasts, providing a more nuanced assessment.

$$\text{CRPS} = \frac{1}{N} \sum_{i=1}^N \int_{-\infty}^{\infty} (P(y \leq t) - 1\{y_i \leq t\})^2 dt, \quad (16)$$

where N is the total number of observations. y_i is the actual value for observation i . $(P(y \leq t))$ is the (CDF) of the predictive distribution at point t . And $1\{y_i \leq t\}$ is the indicator function.

To effectively measure the representation of the temporal correlation structure in the predicted values, we use VarS presented as:

$$\text{VarS} = \frac{1}{L} \sum_{l=1}^L \frac{1}{2(N-l)} \sum_{t=1}^{N-l} \left((\hat{y}_{t+l} - \hat{y}_t)^2 - (y_{t+l} - y_t)^2 \right)^2, \quad (17)$$

where L presents the Number of lags and observations considered respectively. The \hat{y}_t and y_t are the predicted and actual value at time t respectively [52].

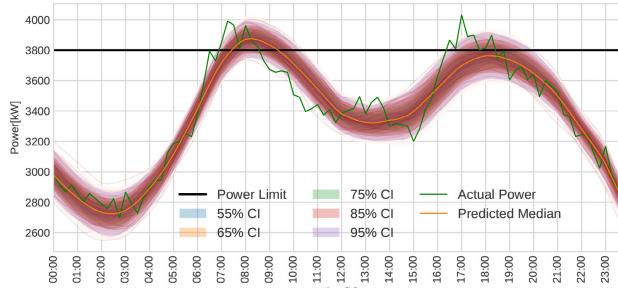


FIGURE 3. A sample day-long hourly AGP-based probabilistic load forecast, indicating uncertainty with different % of confidence interval (Date: DEC 1, 2019) case aggregated loads.

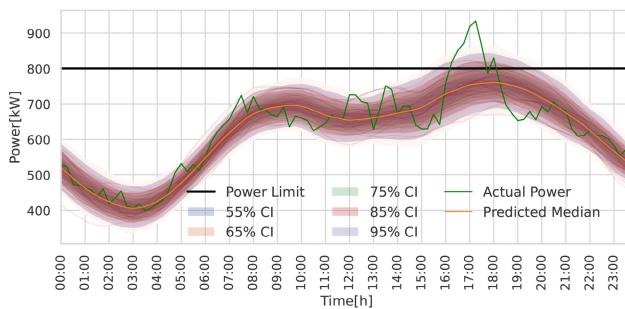


FIGURE 4. A sample day-long hourly AGP-based probabilistic load forecast, indicating uncertainty with different % of confidence interval (Date: DEC 1, 2019) case of aggregated non-flexible loads.

Prophet model forecast: By applying the *Prophet* model with the proposed method, the uncertainties of power

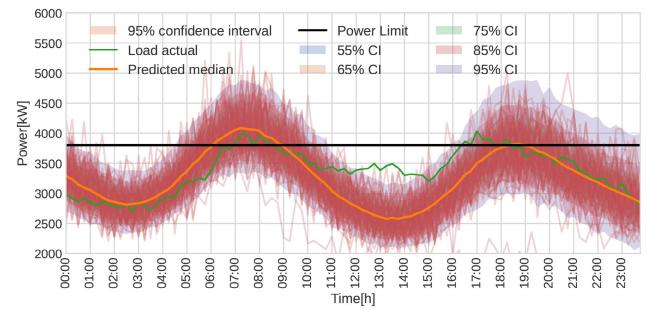


FIGURE 5. A sample day-long hourly Prophet-based probabilistic load forecast, indicating uncertainty with different % of confidence interval (Date: DEC 1, 2019) case of aggregated loads.

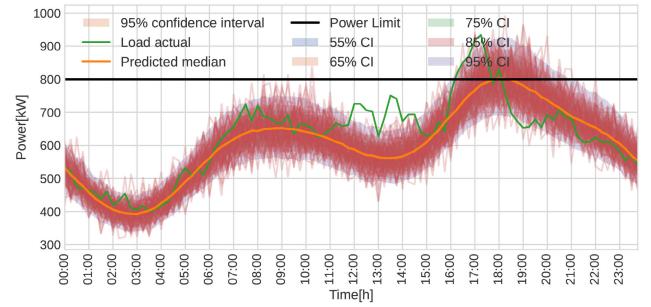


FIGURE 6. A sample day-long hourly Prophet-based probabilistic load forecast, indicating uncertainty with different % of confidence interval (Date: DEC 1, 2019) case of aggregated non-flexible loads.

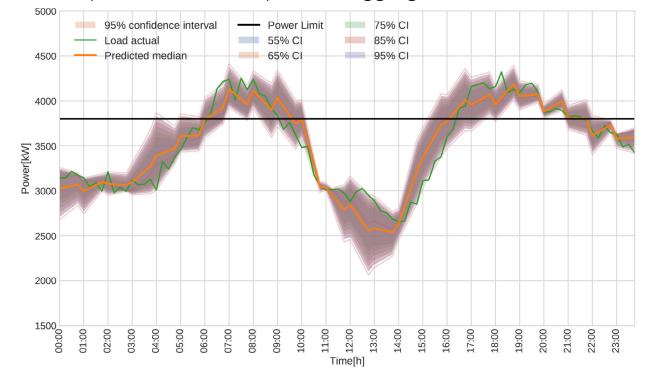


FIGURE 7. A sample day-long hourly QR-based probabilistic load forecast, indicating uncertainty with different % of confidence interval (Date: DEC 1, 2019) case of aggregated loads.

forecasting, and load forecasting with various probability indices (from 55% to 95%) for a day-ahead forecast are represented in Figures 5 and 6 for total aggregated load and aggregated non-flexible load, respectively.

AGP model forecast: Figure 3 shows the hourly forecasting result for the aggregated load of 1000 houses in a 24-hour day-ahead scenario. It elaborates the posterior analysis through AGP to display the degree to which data generated from the model could deviate from data generated from the true distribution. Multiple sample trajectories in Figure 3 visually portray the potential range of load scenarios. The

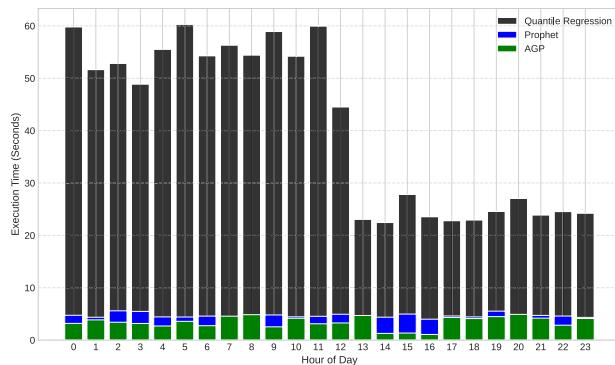


FIGURE 8. Execution time comparative results of all models.

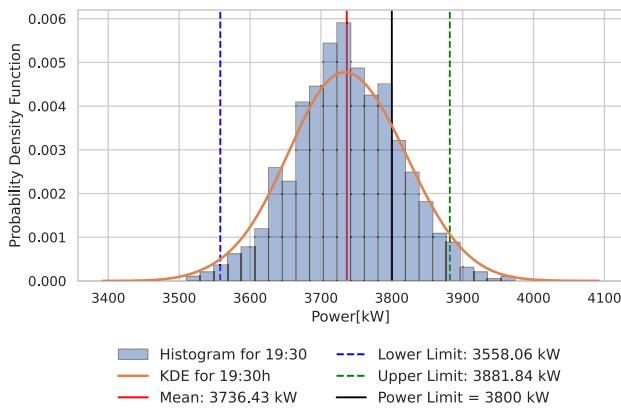


FIGURE 9. Probability density function of the selected hour(19:30) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines with AGP model case of aggregated loads.

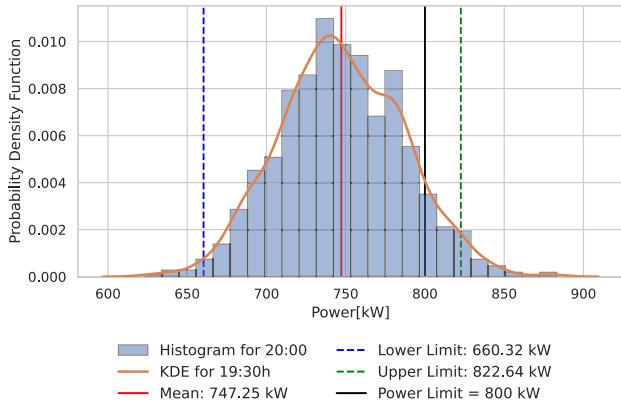


FIGURE 10. Probability density function of the selected hour(20:00) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines with AGP model case of aggregated non-flexible loads.

different confidence intervals ranged between 55%, 65%, 75%, 85%, and 95%, represented by the shaded area around the forecast curve, emphasizing the variability and potential outcomes. This visualization illuminates the inherent

uncertainty in the forecast, offering a detailed perspective on potential load fluctuations within the specified confidence bounds, helping to achieve accurate demand response in the face of uncertainties and variations between predicted and actual electricity consumption. Since our predictive distribution is Gaussian, this quantity enables us to form, for example, a 95% credible set representing the beliefs about the interval, which is 95% likely to contain the truth function compared to the other confidence intervals. As shown in Figure 3, the power uncertainty is higher during the midday period. Conversely, the uncertainty is less during the morning (from 6:00 to 10:00) and afternoon (from 17:00 to 21:00). The depicted Figure 4 represents a day-long hourly probabilistic forecast, focusing specifically on the Aggregated non-flexible loads. The forecasted values are presented as probabilistic distributions, with distribution shapes indicating the forecasted range of heating load values for each hour of the day. Variations in distribution shapes reflect the level of uncertainty in the forecasts, with less sharp distributions indicating higher uncertainty. Various percentiles of the confidence interval represent distinct levels of uncertainty. The power limit is indicated by the black line depending on the requirement for flexibility it is fixed for the aggregated and the Aggregated non-flexible loads to 3800kW and 800kW, respectively. Additionally, it is determined using granular time interval data to mimic the dynamic shape of a customer's demand. It closely follows the actual demand (black line) leading up to and following the event.

As shown in Figure 5, the uncertainty of power forecasting is higher in the middle of the day, when the occupants are at home utilizing more power. While in the morning and afternoon, the uncertainty is less. Power forecasting uncertainty increases and decreases with power increase and decrease, respectively. Also, the uncertainty is increased when the time horizon is larger. For example, at 10:00 and 17:00, power outputs are almost at the same level (about 5000 kW) with higher uncertainty. Figures 5 and 6 illustrate a posterior distribution over a power load variable, which represents the power consumption over time. A solid black line indicates a power limit as a reference, suggesting a constant power level of 2500 kW throughout the depicted period. The shaded regions, delineated by varying levels of transparency, represent different confidence intervals (CIs), such as 95%, 85%, 75%, 65%, and 55%, showing the uncertainty in the power load predictions. A solid black line indicates a power limit as a reference, suggesting a constant power level of 3800kW and 800kW for total aggregated and aggregate non-flexible load, respectively. Additionally, Figures 5 and 6 include two important lines: one in red representing the actual power load observed over time, and another in orange representing the predicted median load. The intersection of these lines with the shaded regions provides insight into the model's accuracy in estimating power consumption at different levels of confidence. These figures provide a comprehensive visual representation of the

TABLE 2. Accuracy metrics for a day ahead forecast with confidence interval 95%.

Strategy	Type Load	MAE[kW]	RMSE[kW]	R ² [%]	sMAPE[%]
Additive Gaussian Process	Total Load	203.00	240.04	0.71	8.27
	Heating Load	141.03	161.05	0.73	15.36
	Other Load	32.61	43.89	0.83	5.19
Prophet	Total Load	266.47	355.78	0.69	9.59
	Heating Load	230.65	238.19	0.60	24.64
	Other Load	49.81	68.50	0.72	7.69
Quantile Regression	Total Load	234.25	295.98	0.69	8.40
	Heating Load	57.17	100.49	0.90	11.21
	Other Load	69.09	85.69	0.52	6.77

TABLE 3. Probabilistic accuracy metrics.

Strategy	Type Load	PICP	CRPS	VarS
Additive Gaussian Process	Total Load	0.82	569.02	15.94
	Heating Load	0.86	639.81	17.10
	Other Load	0.95	183.73	15.03
Prophet	Total Load	0.80	963.32	16.46
	Heating Load	0.96	1647.69	16.48
	Other Load	0.94	253.88	16.30
Quantile Regression	Total Load	0.83	944.35	16.42
	Heating Load	0.93	267.49	17.02
	Other Load	0.83	273.66	16.61

uncertainty associated with power load predictions and a comparison to the actual observed load.

Quantile regression model forecast: Figure 7 shows a day-long hourly forecast for aggregated load using the quantile regression model. For the brevity of the presentation, the quantile regression is displayed only for the aggregated load. However, Tables 2 and 3 depict the accuracy metrics using quantile regression for all the cases of total load, heating load, and other load. The model is based on quantile regression and then enhanced with a temporal dependence structure. A semi-parametric methodology for generating such densities is presented; it includes a time-adaptive quantile regression model for the 5%-95% quantiles [41], [53]. The accuracy of these models is assessed for various load types, with the results summarized in Tables 2 and 3. The average sMAPE for 1-day forecasting horizons for different load types is found to be 8.27%, 15.36%, and 5.19%, respectively. VarS of the proposed method for different load types is found to be 15.94, 17.10, 15.03, respectively. Importantly, the AGP model outperforms the *Prophet* model

and quantile regression when applied to the aggregated total load and the aggregated non-flexible loads, respectively. This work underscores the variation in electricity consumption forecast based on different load types and the importance of considering weather and calendar variables in peak load demand forecasting. The AGP model demonstrates superior performance in short-term load forecasting with uncertainties, offering valuable insights for grid management and flexibility analysis as compared to the *Prophet* model. Moreover, the execution time (Figure 8) also shows the superiority of the proposed method.

3) UNCERTAINTY QUANTIFICATION AND FLEXIBILITY DEMAND CALCULATIONS

The statistical analysis to assess the forecasted uncertainty and flexibility calculations can be leveraged by DSOs to evaluate the system security level or assess the demand flexibility for the considered day. Specifically, in the flexibility markets, it can often be used to manage consumption by setting capacity limit thresholds or limiting power consumption [54].

Hence, extracting uncertainty distributions for specific times, guided by selected confidence intervals by establishing a power limit, such as 3800 kW for 1000 households, becomes pivotal in making decisions to adjust consumption. The hourly probability distribution obtained from the forecast errors represents the likelihood of power consumption being less than or equal to a specific value in kilowatts (kW). Statistically, the Probability Density Function (PDF) can help determine the appropriate threshold based on the desired level of risk.

The PDFs provide a visual tool for assessing the likelihood of extreme events to perform risk assessment [27]. This section provides the discussions related to the uncertainty quantification and flexibility calculations by effectuating the proposed methodology with two employed models, namely AGP and *Prophet* forecasts. Additionally, a comparative analysis is achieved by plotting the hourly distribution of the load forecasts resulting from two models for identifying trends and anticipating the flexibility needs. Since the samples are taken from the posterior distribution, their probability density also needs estimation. Hence, we use Kernel Density Estimation (KDE) to achieve the nonparametric probability density, where we center a smooth scaled kernel function at each datapoint and then take their average [55]. Note that it is an empirical distribution that cannot be expressed analytically. The forecasting uncertainty can be represented as upper and lower-bound margins around the power forecast. The probability density can be drawn according to the selected samples from the predictive analysis since the forecast errors can be expressed as a percentage of the rated power. The bound margins are extracted from an inverse cumulative distribution function [27]. By CDF, we denote the function that returns probabilities of aggregated power y bounded lower to a value y_α , i.e.,

$$\text{prob}(y \leq y_\alpha) = F(y_\alpha), \quad (18)$$

where α is the desired percentile. Now, the inverse of the CDF gives a value y_α for which the $F(y_\alpha)$ will return α , i.e.,

$$F^{-1}(\alpha) = y_\alpha. \quad (19)$$

From (18) and (19), we can get surpassed aggregated power (flexibility requirement) by which it exceeds the power limit (y_{lim}), i.e. $\Delta y = y_\alpha - y_{\text{lim}}$.

AGP Forecasting: Figures 9 and 10 depict the probability distribution at a particular hour for aggregate total load and aggregated non-flexible load for the forecast resulting from the AGP forecasting model. The kernel density estimated value is represented by the orange line, reflecting a specific power at that point and representing how the PDF values change across the band. It comprehends data distribution, sets thresholds, evaluates risks, and calculates how much the aggregated power consumption can cross the upper capacity threshold. The upper and lower bounds are defined at 95% confidence levels of the aggregated power consumption. This confidence level quantifies the associated uncertainty of power values. This visualization aids in understanding the

probability distribution of power values and the influence of confidence levels. In Figure 9, the visualization for a one-day ahead at 19:30h with the upper bound of power 3881.55kW and the lower bound of power 3558.06kW is shown to exceed the power limit (y_{lim}) fixed to 3800kW by 81.55kW. Note that in this case study, the power limit is established at specific values: 3800 for the scenario involving an aggregated load of 1000 houses and 800 for the scenario concerning non-flexible load. A similar analysis is carried out in Figure 10 for the non-flexible loads.

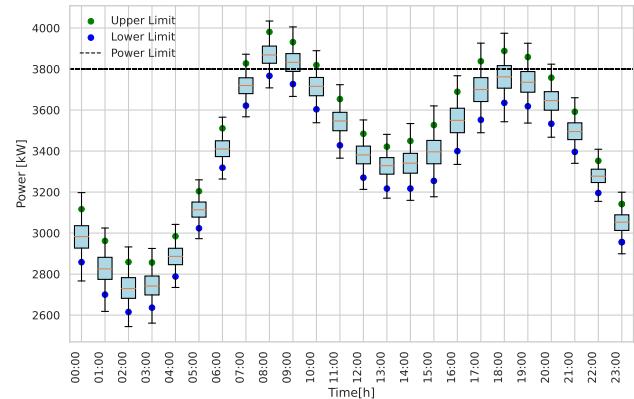


FIGURE 11. Hourly uncertainty based on a day-ahead forecast of aggregated loads from AGP-based method.

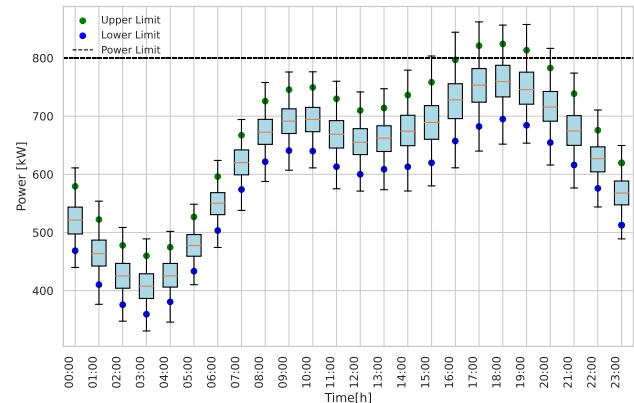


FIGURE 12. Hourly uncertainty based on a day-ahead forecast of aggregated non-flexible loads from AGP-based method.

Figures 11 and 12 illustrate uncertainty quantifications resulting from the AGP forecasted results for every hour in a 24-hour day-ahead scenario. It can be observed from Figure 11 that the capacity threshold is crossed during the peak hours of a typical day, i.e., from morning 7:00 to 10:00 a.m. and from 5:00 to 8:00 p.m. For the aggregated non-flexible load (Figure 12) the duration of crossing the capacity threshold is limited to the evening hours, indicating the heavy use of non-flexible loads during this period.

Prophet and quantile regression forecasting: Similar to the uncertainty analysis performed for the results for the AGP

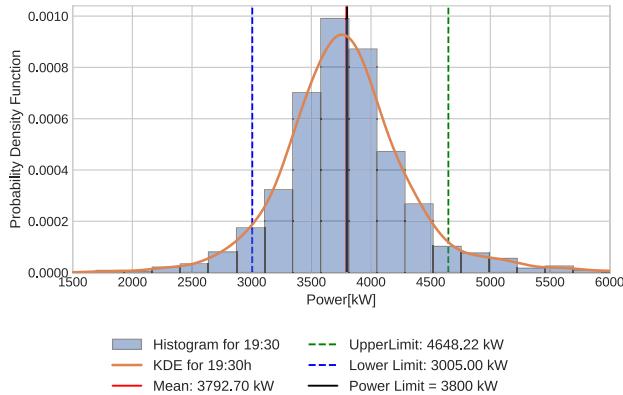


FIGURE 13. Probability density function of the selected hour(19:30) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines from the *Prophet*-based model in case of aggregated loads.

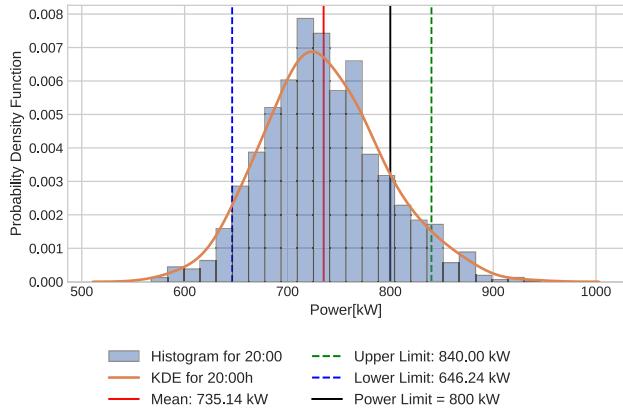


FIGURE 14. Probability density function of the selected hour(20:00) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines from the *Prophet*-based model in case of aggregated non-flexible loads.

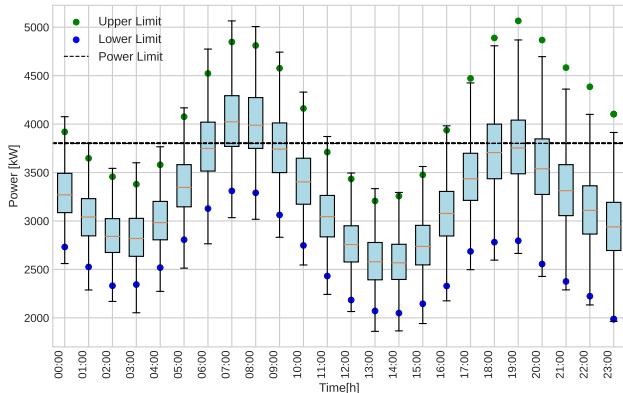


FIGURE 15. Hourly uncertainty based on a day-ahead forecast of aggregated loads from the *Prophet*-based method.

model forecast, Figures 13 to 16 correspond to the forecasting results from *Prophet* model. Additionally, Figures 17 and 18 correspond to the forecasting results from quantile regression

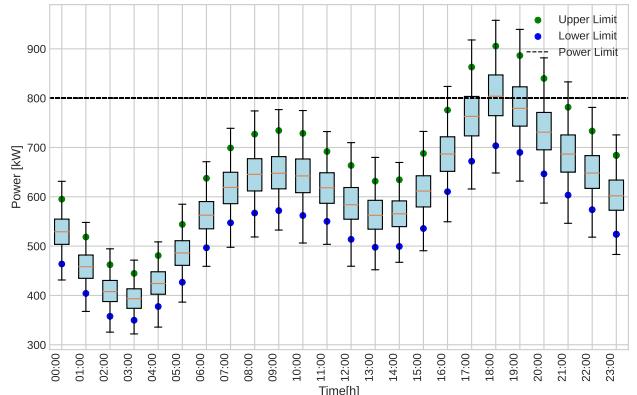


FIGURE 16. Hourly uncertainty based on a day-ahead forecast of aggregated non-flexible loads from the *Prophet*-based method.

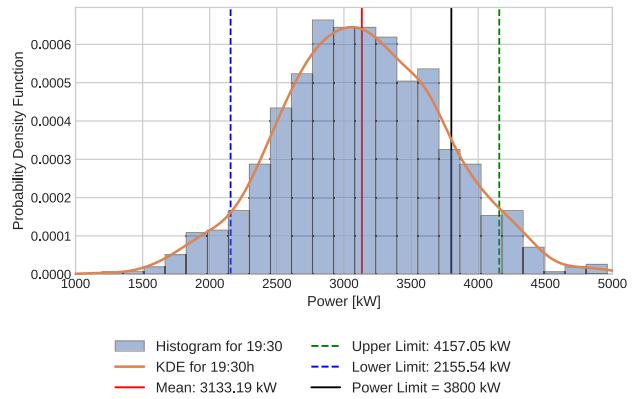


FIGURE 17. Probability density function of the selected hour(19:30) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines with Quantile Regression model case of aggregated loads.

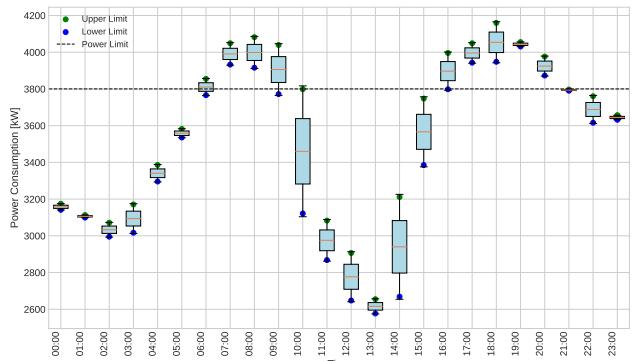


FIGURE 18. Hourly uncertainty based on a day-ahead forecast from Quantile regression model case of aggregated loads.

for aggregated total load case. Comparative results for AGP and Prophet models are depicted in Figure 19 and 20 for aggregated total and aggregated non-flexible loads, respectively. Note that for comparative analysis, both the models are trained on the same set of synthetic data of

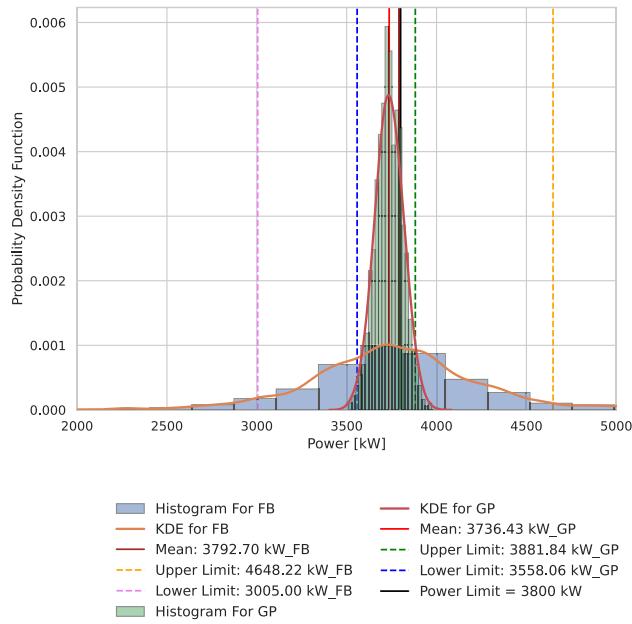


FIGURE 19. Probability density function of the selected hour(19:30) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines for aggregated loads.

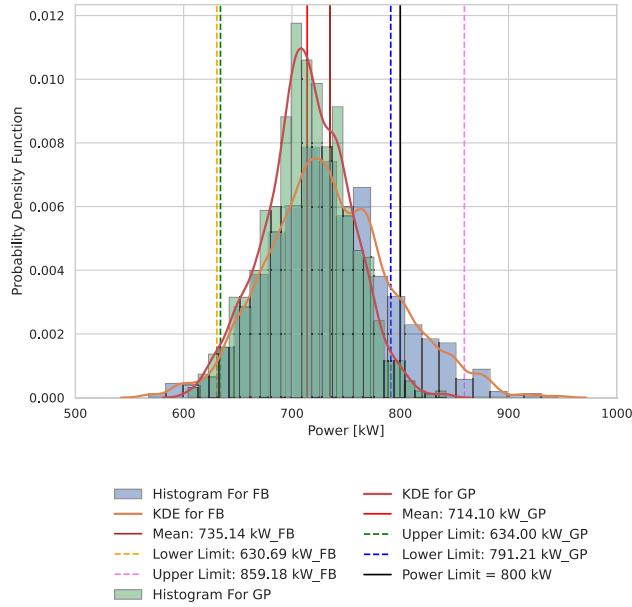


FIGURE 20. Probability density function of the selected hour(20:00) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines for aggregated non-flexible loads.

1000 houses. The PDFs are plotted for the hour 19:30 for the case of aggregated load and 20:00 for the non-flexible load of peak usage and the quantified uncertainty for the same day-ahead predictions is displayed. It is clear that the flexibility needs prediction resulting from the AGP-based method is much lower compared to the *Prophet* and quantile

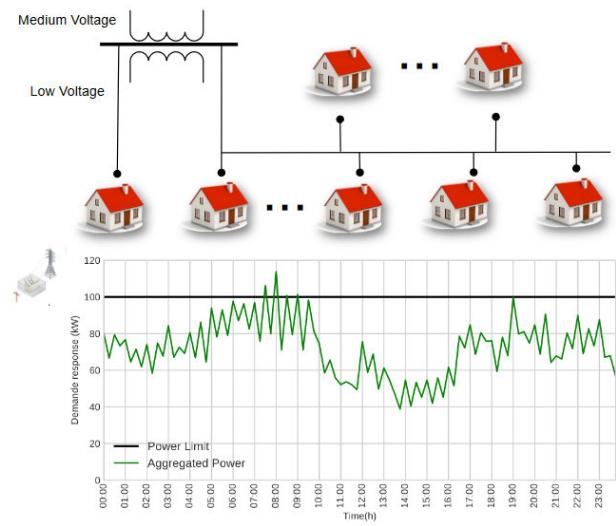


FIGURE 21. Load profile of 14 households in the peak morning and evening hours on Dec. 21, 2018, connected to the 100 kVA distribution transformer.

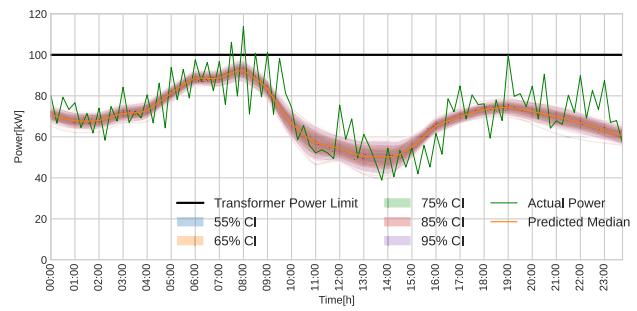


FIGURE 22. Predicted aggregated end-user load (solid orange line) and associated uncertainty obtained using the AGP on a typical winter day.

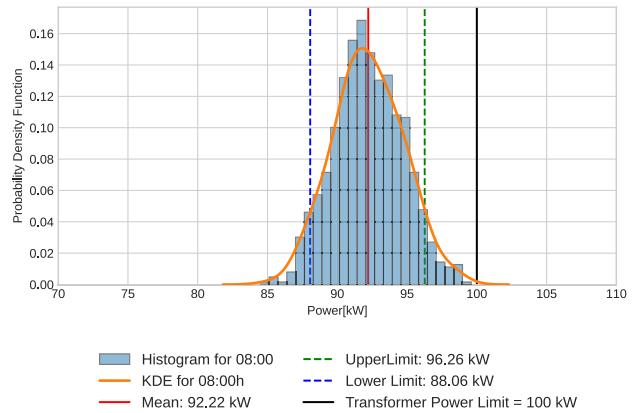


FIGURE 23. Probability density function of the selected hour (08:00) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines for aggregated load.

regression model-based results. This indicates the superiority of the AGP-based methodology in short-term load forecasting

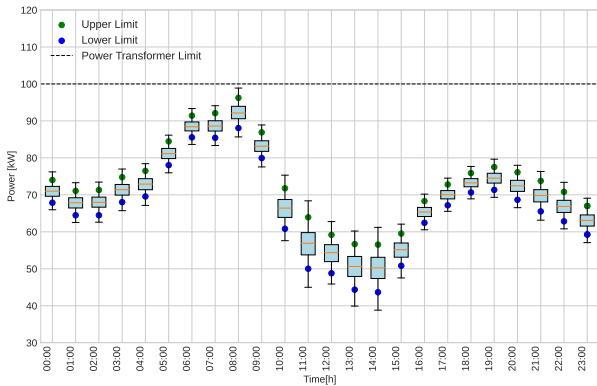


FIGURE 24. Hourly uncertainty based on a day-ahead forecast of aggregated loads from the AGP-based method.

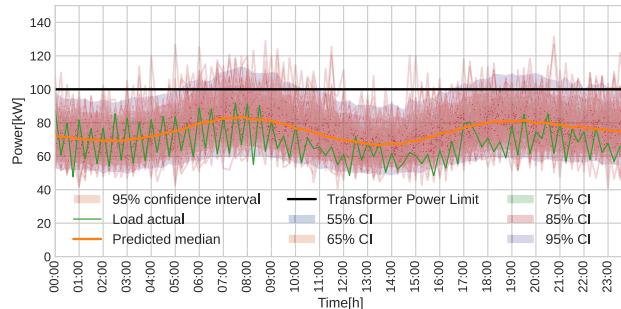


FIGURE 25. Hourly uncertainty based on a day-ahead forecast of aggregated loads from the Prophet-based method.

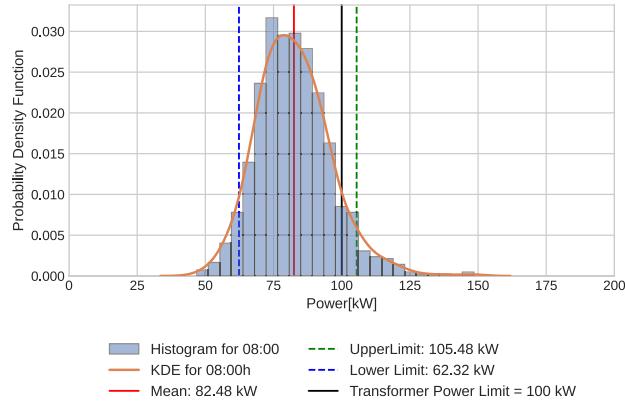


FIGURE 26. Probability density function of the selected hour (08:00) using both a histogram and a kernel density estimate. The upper and lower limits of power at 95% confidence levels, as well as a power limit value, with vertical lines for aggregated load.

with uncertainties, which results in precise uncertainty quantification.

B. CASE STUDY ANALYSIS OF 14 AGGREGATED HOUSES

To demonstrate the performance of the proposed approach for effective demand response decision-making, we assume a case study to forecast the aggregated power consumption of

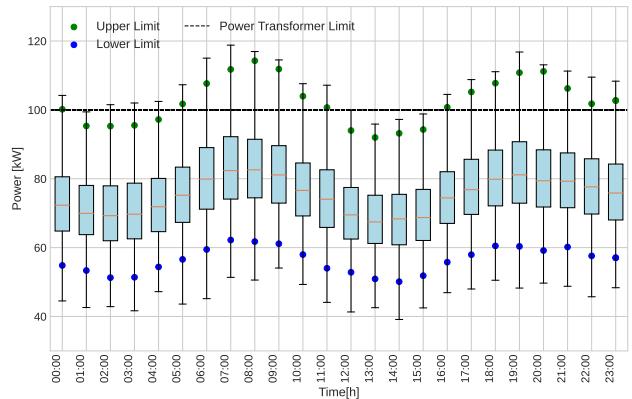


FIGURE 27. Hourly uncertainty based on a day-ahead forecast of aggregated loads from the Prophet-based method.

14 households supplied by the same low-voltage transformer, assuming the threshold capacity of 100 kW. In this case study, the 14 consumers were randomly selected from a database of 1000 houses. Three of the houses in this group consume 10 kW each, while the rest consume between 5 and 10 kW. This study considers the peak period in winter December 2018. During peak usage periods, the network's transformer capacity is critical. To avert overloading, particularly in cold weather conditions, the aggregator could anticipate the fluctuations in demand using the forecasted power consumption to instruct and encourage consumers to adjust their usage patterns in response to changes in electricity prices, grid conditions, or environmental concerns, and regulate their electricity consumption according to the existing flexibility scope. Figure 21 illustrates the aggregated power profile of 14 households on December 21, 2018, where the black line denotes the maximum power limit (y_{lim}), set at 100 kW. To streamline our analysis and avoid repeating figures for all scenarios, we focus specifically on the aggregated load forecast analysis using AGP and *Prophet* models. Beginning with the forecasting of the first model and following the previously mentioned methodology, the uncertainty analysis results for the case of 14 houses are determined. *Forecast and uncertainty analysis using AGP:* The assessment of uncertainty in demand forecasting was conducted using the hourly probability density function derived from the forecast errors. Subsequently, the hourly risk curve was developed by incorporating all errors from these PDFs. Given that forecast errors can be quantified as a percentage of the rated power, the power consumption can be established based on the inverse cumulative function. Figures 22 and 23 illustrate the necessary variations, following the methodology outlined in Section II. The result indicates that the uncertainty associated with its data predictions remains within the aggregated power limit set for the specified transformer and does not exceed the upper power limit.

Forecast and uncertainty analysis using Prophet: Conversely, with the *Prophet* model, we observe (Figures 25,

26 and 27) an approximate additional load (surpassed power (Δy) of 5 kW. This value represents the flexibility requirement, as determined within the 95% confidence interval. In essence, this additional load reflects the extra capacity that the system might require to handle unforeseen fluctuations, ensuring reliability and stability in power supply to the group of 14 houses under study. As demonstrated in Figures 24 and 27, there are noticeable differences in peak load predictions during morning and afternoon periods between the two models. This analysis highlights a dual perspective: Firstly, compared to the Prophet method, the advanced forecasting approach with AGP yields superior forecasting results for the short-term horizon. Secondly, better forecasting by AGP lays a robust foundation for strategic planning. That enables organizations to craft flexible, well-equipped strategies to handle various future scenarios. Applying this approach in a real-world case study with real-grid constraints can help create an efficient indicator for decision-making related to demand response and energy consumption, allowing energy utilities to have proactive communication with consumers ahead of time about the expected demand response events based on forecasted demand.

V. CONCLUSION

This study introduces a method for quantifying the uncertainties and calculating the flexibility needed for aggregated household power consumption forecasts. The methodology utilized the AGP approach to perform short-term load forecasting by considering uncertainties. A statistical investigation was conducted to quantify these uncertainties on an hourly basis, leading to flexibility need calculations for peak hours when the power load is most susceptible to exceeding capacity limits. For comparative analysis, the Prophet forecasting model and classical quantile regression were also used to perform forecasting and quantify uncertainties. The investigation was applied to a synthetic dataset comprising 1000 residential buildings in Québec, Canada, with a fixed prediction window of 24 hours ahead. Additionally, a case study involving 14 households connected to the same transformer was conducted. The results demonstrated the superior accuracy of AGP-based forecasts, with more precise prediction and better hourly uncertainty and flexibility requirement calculations compared to the *Prophet* model and quantile regression. This work enhances grid capacity limitation services by improving forecasting accuracy, thereby supporting more informed decision-making. Future research may explore detailed consumer behavior modeling to better forecast energy demand and capture the variability and complexity of energy consumption across different users.

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