A Clearing Mechanism with Reduced Computational Complexity for Spot Flexibility Markets

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***Abstract*—The spot flexibility markets are before the real-time energy exchange, allowing demand-side management to reduce energy consumption during peak periods. In these markets, demand aggregators must quickly choose the customers' reduction bids that fulfill grid requirements. This clearing procedure is challenging due to the computational complexity of selecting the optimal bids. Therefore, developing a clearing mechanism that avoids searching the entire flexibility bid space while respecting grid constraints is essential for the smooth operation of the spot flexibility market. This paper presents a clearing mechanism with reduced computational complexity of the winner determination problem in spot flexibility market for demand aggregators carrying out reductions in energy consumption. The proposed approach transforms customers' flexibility bids into a reward-based function. Afterward, the gradient-based optimization solves the bid selection problem. This approach helps demand aggregators achieve satisfactory energy reductions within an appropriate delay for spot flexibility markets. A comparative study presents the effectiveness of the proposed approach against commonly used approaches: hybrid particle swarm optimization genetic algorithm and combinatorial search.**

Index Terms—Combinatorial auction, computational complexity, demand response, flexibility, spot flexibility market, transactive energy, clearing, gradient-based optimization.

I. Introduction

THE development of various congestion management (CM) strategies in the distribution network context aims to address situations where electricity demand exceeds the grid capacity [1]. These scenarios can overload crucial grid elements like transformers and distribution lines, potentially increasing costs, leading to power outages, and decreasing overall system efficiency [2]. In cold geographical regions, the increase in the electricity heating load consumption and the proliferation of electric vehicles (EVs) further intensify these situations [3]. CM approaches to tackle such issues can be grouped into direct-control, price-based, and market-based ones [1], [4]. The direct-control approach enables the grid operator to have uninterrupted access to control customers' specific loads, which involves deliberately cutting off power, reducing power consumption, or rerouting power via alternate pathways [5]. Price-based approaches include dynamic pricing strategies, such as time-of-use tariffs, encouraging customers to modify their energy consumption patterns based on priced signals and reducing electricity demand during peak congestion periods [6]. The market-based approach uses economic incentives to minimize the difference between electric supply and customers' demand [7]. Moreover, establishing flexibility markets incentivizes consumers to reduce or shift their energy usage during peak periods, thus alleviating congestion [8]. Traditional grid reinforcement strategies are expensive and require extensive planning [9]. Distribution system operators (DSOs) are exploring innovative solutions to address CM, including integrating information and communication technology (ICT) and establishing new markets for obtaining flexibility [10].

The transactive energy framework (TEF) empowers participants within their energy management systems and facilitates bidirectional information exchange with grid operators for grid balancing using economic signals [11]. The TEF application in distribution networks can address challenges like voltage management, CM, and the establishment of new electricity spot flexibility market (SFM) mechanisms to engage participants with flexible loads [12]. Furthermore, TEF enables demand aggregator agents to represent groups of customers capable of providing flexibility. In this paper, the focus is on SFM, where residential customers participate to obtain immediate trades. Regarding the SFM structures, aggregator agents can use auctions, allowing residential customers to present flexibility bids [13]. The most common types of

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electricity auctions are double-sided and single-sided. In the former, there are many electricity-providing agents and many consumers, whereas, in the latter, there is a single electricity-providing company and many consumers [14]. In certain geographic regions, a single utility company is responsible for electricity generation, transmission, and distribution [15]. In this case, the option is to implement single-sided auctions, for consumers to interact with the utility company [16].

In single-sided auctions, combinatorial search is the conventional approach to clearing the market. It guarantees the best selection of bids that fulfill the flexibility request from the DSO. However, it has a high computational complexity because the search space increases drastically with the number of participants. The aggregator agents must examine every potential combination to identify residential customers' bids that meet the grid requirements. The bid selection in a combinatorial single-sided auction (CSA) is known as the winner determination problem (WDP) [17]. WDP falls into an non-deterministic polynomial hard (NP-hard) problem [18], which is challenging for the aggregator to solve quickly. Therefore, most WDP algorithms are impractical for SFMs.

The relevant literature includes the research works employing flexibility markets to manage congestion, along with studies that explore mixed-integer linear programming (MILP), machine-learning-based approaches, and meta-heuristic-based techniques for clearing the auction-based market.

1) Flexibility Market

Reference [19] proposes a transactive energy-based flexibility market between aggregators and the DSO for CM. Aggregators control consumer loads as a single virtual storage to maximize profits, which reduces consumer control. Reference [7] proposes a distribution-level flexibility market to address contingencies resulting from day-ahead energy markets. Nevertheless, unexpected real-time events can increase DSO costs. In real-time CM, a market-based scheduling framework is proposed to reduce computational costs [20]. Consumers submit their load curtailment ratios and EV charging tolerances to the aggregator. However, this scheduling may limit user flexibility in adjusting their charging plans. In [21], consumers with their energy management systems employ backward induction within adaptive dynamic programming to submit flexibility bids based on locational price of DSO for CM. The DSO employs Taylor series approximation to linearize problem constraints and conducts the optimal power flow for price and power dispatch.

The DSO collaborates with aggregators to implement a single-sided auction market negotiation framework for CM, using a uniform pricing mechanism for market clearing [22]. Aggregators directly control consumer loads to offer flexibility services. However, the aggregators do not consider the negotiation with consumers, who are the actual flexibility providers. In real-time CM, the aggregator collaborates with the DSO to control user loads and coordinate load swaps with other aggregators [23]. The DSO compensates based on the number of swaps made, yet the aggregator overlooks customer willingness to provide flexibility in these swaps. Consumers control their loads in the real-time hourly market for flexibility services and submit bids for energy reduction [24]. The aggregator then aggregates and sorts these bids based on price. Only those bids below the market clearing price are selected and those above it are excluded, even though consumers are willing to offer flexibility. In [25], a transactive energy-enabled local flexibility market (LFM) is proposed. Consumers submit bids to an LFM aggregator. The LFM aggregator sends the bids to a central market aggregator for clearing, which clears the market using a mixed-integer program and distributes rewards. Accepted bids receive rewards based on the market clearing price. However, those with unaccepted bids receive no reward. Within a flexibility market, aggregators manage consumer load to submit flexibility bids to the local market operator for CM [26]. Nevertheless, details are not provided regarding the specific market clearing algorithm the local market operator utilized for handling these flexibility bids of aggregator.

2) MILP Approaches

In [27], a game theoretical model for load shaving undergoes reformulation as a single-level MILP, aiming to optimize retailer profits by acquiring flexibility from consumers in a demand response market. Still in [27], involving more consumers increases the number of integer variables and associated constraints, resulting in a substantial computational burden, and the approaches consider only three consumers. Table I shows the comparison of an auction-based market clearing framework.

TABLE I
COMPARISON OF AN AUCTION-BASED MARKET CLEARING FRAMEWORK

Ref.	Objective	Auction between consumers and aggregator	Market clearing time	*XOR* bid	All consumers assigned	Avoid direct load control	Scalable larger than 50 consumers	Stability
[28]	Avoiding power outages	√	Spot market	×	×	√	√	×
[29]	Providing capacity in energy storage	√	Day ahead	√	√	√	×	×
[30]	Reducing load by providing incentive	√	Day ahead	√	√	√	×	×
[31]	Minimizing energy trading cost	√	Day ahead	√	√	√	×	×
[19]	CM using EV flexibility	×	Spot market	×	×	×	√	√
[22]	CM in distribution network	×	Spot market	×	×	×	√	√
This paper	Providing flexibility to DSO	√	Spot market	√	√	√	√	√

A stochastic MILP has been formulated in [32] to enable communities with electric heat pumps to provide demand-side flexibility. The approach demonstrates robustness in managing congestion while ensuring consumer comfort does not fall below agreed-upon limits during these events. However, the formulation is not suitable for SFM due to its computational complexity. Reference [33] proposes an MILP-based optimization framework for a demand aggregator for flexibility provision by penalizing customers. However, the specific usage constraints of individual appliances and the uncertainties arising from consumer behavior are not considered.

3) Machine-learning-based Approach

Reference [34] proposes a machine-learning-based approach to help the aggregator profit maximization by choosing bids from residential consumers within a combinatorial auction, ultimately facilitating the market clearing process. Moreover, in [35], a multi-agent reinforcement learning (RL) framework is utilized for efficient local energy trading in a 15 min continuous auction-based market. This approach offers quick training but requires careful hyperparameter tuning.

Reference [36] proposes a deep RL-based auction energy market, validated through simulations showcasing the profitability for all participants. However, the approach does not consider the solution within a continuous action space. A recent review paper summarizes the application of machine learning algorithms for addressing combinatorial auctions in the power system and suggests that there are still challenges related to scalability and accuracy in real implementation scenarios [37]. The machine-learning-based approaches aim to emulate the optimization problem offline. However, they encounter difficulties in re-training when the auction scenario or the number of participants changes.

4) Meta-heuristic-based Techniques

According to an in-depth review presented in [38], utilizing computational intelligence techniques for optimization in local electricity markets primarily entails meta-heuristic-based techniques, such as particle swarm optimization and genetic algorithms. These approaches effectively address complex optimization problems, but they require a high number of iterations and the obtained solutions are frequently subject to variability due to the stochastic nature of the search process. A recent literature review in [39] has delved into market clearing mechanisms in local flexibility markets and emphasized the ongoing challenge of computational complexity as a critical limitation in the field.

Reference [40] proposes a multi-layer ant colony optimization algorithm for scheduling energy resources and minimizing operation costs in standalone microgrids through single-sided auctions within a local energy market. As reported in [41], with an extensive literature review, the computational complexity remains a significant obstacle to implementing these programs in real-world scenarios. According to [42], meta-heuristic-based techniques deal with computational complexity but jeopardize optimality. An algorithm for solving combinatorial auctions using a constraint-guided evolutionary approach is presented in [43], specifically for the combinatorial reverse auction of power generation and transmission line assets. References [28] and [30] present closely related studies to the present work, addressing peak electricity demand management through combinatorial reverse auctions. Reference [44] proposes an algorithm that utilizes Taylor series approximation to reduce the computational complexity for the aggregator to maximize its profit in a combinatorial single-sided auction (CSA). In all these cases, meta-heuristic-based techniques or approximation algorithms are employed to reduce the computational time from exponential to polynomial. Table I compares the present work with similar literature. Some known limitations of these state-of-the-art works are as follows.

1) With the meta-heuristic-based techniques, the iterations required to achieve the optimal solution increase as the number of participants increases. This condition results in slow clearing mechanisms, which may exceed the short duration of SFM.

2) With each iteration, the WDP solution may have a significant variance due to the stochastic nature of the search in some meta-heuristic-based techniques.

Considering the state-of-the-art single-sided auctions in SFM, this paper presents a clearing mechanism to deal with existing limitations. In particular, the proposed approach reduces computational complexity for demand aggregators to solve WDP and provides a deterministic solution. The main contributions of this paper are summarized as follows.

1) The proposed approach enables the spot flexibility market aggregator (SFMA) to reduce the computational complexity of a CSA and quickly respond to demand reduction requests from the DSO during peak periods.

2) The solution attains consistency by formulating the discrete bids to a continuous function and solving through a gradient-based interior-point optimization approach for the combinatorial auction process, eliminating solution variance.

3) The proposed approach aids the SFMA in maximizing its profit by choosing the customers' bids that approximate the best payoff.

The subsequent sections are organized as follows. The SFM model is shown in Section II. The proposed WDP is presented in Section III, along with grid constraints. Section IV is dedicated to the simulation results. Finally, Section V presents the concluding remarks.

II. SFM Model

Reference [45] outlines various business models addressing congestion issues through DSO-aggregator interaction. These models include isolated, tariff-based, iterative, and market-based models for distribution CM. This paper adopts a market-based model because it yields higher profits for the SFMA [45]. It is considered that the residential agents (RAs) control customers' flexible loads and interact with the SFMA in the market-based model, as shown in Fig. 1 [30]. In the planning procedures, the RAs report their forecasted energy demand for a defined period. The forecast constitutes a consumption baseline for the DSO to plan grid operation. Then, during peak periods, the SFMA transmits the set of different reward points before the real-time energy exchange starts, and the RAs generate energy reduction bids considering each possible reward. As mentioned, this procedure is a CSA. Each RA transmits its bids using an *XOR* bid language, meaning the SFMA can choose only one option from the bid set. Here, the SFMA has two objectives: maximize its profit and fulfill the demand reduction request. Notably, the request comes from the DSO ahead of the market period.

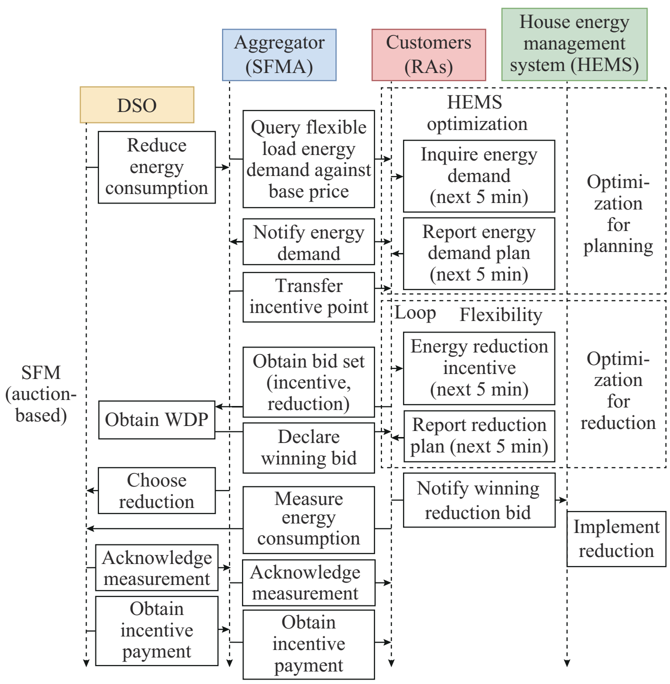

Fig. 1. SFM aggregator interaction with DSO and RAs in unified modeling language.

A. DSO

The DSO manages the distribution network and guarantees its reliability, security, and adequacy. The DSO supplies energy to consumers and purchases flexibility from them through an aggregator agent. Additionally, it plays a vital role in monitoring and measuring the energy consumption within the distribution network. The DSO triggers a congestion event alarm whenever the peak power demand of the consumer overpasses the grid limits. Overloaded distribution lines are a barrier to reliable energy flow in low-voltage networks. To solve this problem, the DSO sends a demand reduction request to the SFMA serving the specific congested circuit. The DSO can encounter two types of costs: either operational or transactive. Operational costs for CM involve purchasing expensive energy from neighboring sources or running costly peak power plants, denoted by C_o [46]. Transactive costs include the monetary compensation of DSO ξ_k to the SFMA, derived from a portion of energy sale profits, where $\xi_k < C_o$ [46].

B. SFMA

The aggregator agent acts as a flexibility provider to the DSO and an intermediary for the RAs. The SFMA is responsible for a particular customer group in a defined geographical area. Indeed, it behaves as the auctioneer in that local SFM. Once the SFMA receives the load reduction request from the DSO for the next time slot, it requests all participating RAs to report their energy demand plan or consumption baseline, as illustrated in Fig. 1. This estimate gives the probable energy demand for the next time slot. Afterward, the SFMA initializes the single-sided auction to procure flexibility from the RAs and attain the reference energy reduction requested in the next time slot. The most common approaches for price settlement among participants in auctions are pay-as-clear (PAC) or pay-as-bid (PAB) [47]. In the former, all participants receive the market clearing price when supply and demand reach equilibrium, while in the latter, all participants receive the price according to their offered bid [47]. In the context of the flexibility market, a suggested approach for managing congestion in distribution networks is PAB [48]. The non-homogeneous nature of flexibility bids leads to the high fragmentation of the flexibility product in markets [48]. Clearing these bids through the PAC approach could increase the cost of flexibility for the DSO [49]. The PAB approach motivates participants by ensuring that they receive gains corresponding to their willingness to reduce consumption.

The auction process starts by sending the bidding request and the set of reward points to all the RAs. The SFMA, as auctioneer, sets the duration of the reduction bids. According to the literature, it is beneficial to use 5 min time slots [50]. Once the SFMA gathers all offers, it starts the WDP algorithm. The CSA-based winner determination results in a single bid selection for each participant. Next, the SFMA transmits the energy reduction outcomes to all participants for the next time slot and terminates the single-sided auction. The SFMA also sends auction results to the DSO to validate the demand reduction.

C. RA

Each RA participating in the SFM has an HEMS capable of controlling its flexible loads. The set of RAs is denoted as RA^i $(i = 1, 2, ..., N)$, with N being the total number of participants. We consider that these customers have thermostatically controlled loads (TCLs), which allow them to shift energy consumption in time. In the time slot k of the SFM, the state-space model of the TCL relates indoor temperature T_k^i with energy consumption e_k^i, as shown in (1) [51]. This model also relates the outdoor temperature $T_k^{out,i}$. The coefficients β_z, β_e, and β_o represent heat dissipation or absorption efficiencies.

$$T_k^i\left(e_k^i\right) = \beta_z T_{k-1}^i + \beta_e e_k^i + \beta_o T_k^{out,i} \tag{1}$$

Each RA in the SFM exhibits different preferences towards energy reduction [52]. The parameter P_k^i expresses the preferences for maintaining comfort by remaining close to the set point T_{sp}^i or earning the rewards by deviating from the set point. The benefit for each residential consumer B_k^i is quantified according to its preferences as:

$$B_k^i\left(e_k^i\right) = P_k^i\left(T_k^i\left(e_k^i\right) - T_{sp}^i\right)^2 \tag{2}$$

During the peak period, the aggregator asks the residential consumer to report the forecasted energy demand $e_{k+1}^{i,base}$ for the next 5 min time slot, considering a base price of λ_{base}^i.

Thus, each residential consumer solves the optimization problem in (3) to maximize its utility.

$$\begin{cases} \max\limits_{e_k^{i,base}} \left(B_k^i\left(e_k^{i,base}\right) - \lambda_{base}^i e_k^{i,base} \right) \\ \text{s.t. } 0 \le e_k^{i,base} \le e_{\max}^i \\ \quad T_{\min}^i \le T_k^i\left(e_k^{i,base}\right) \le T_{\max}^i \end{cases} \tag{3}$$

where $e_{\max}^i$ is the installed energy capacity; and $T_{\max}^i$ and $T_{\min}^i$ are the upper and lower indoor temperature bounds, respectively.

Afterward, the SFMA sets up the auction for purchasing flexibility from the RA. The SFMA transmits the set of incentives λ_j^i and requests the RA to provide the corresponding reduction offers. This incentive set is represented by $\lambda_j^i := \{\lambda_j^i, j = 1, 2, ..., M\}$ including M incentive points. Then, the RAs recalculate by (4), considering each reward to estimate the corresponding energy reductions $e_{k,j}^{i,red}$. Consequently, the forecasted indoor temperature for the next time slot is also recalculated.

$$\begin{cases} \max\limits_{e_{k,j}^{i,red}} \left(B_k^i\left(e_k^{i,base} - e_{k,j}^{i,red}\right) + \left(\lambda_j^i + \lambda_{base}^i\right) e_{k,j}^{i,red} - \lambda_{base}^i e_k^{i,base} \right) \\ \text{s.t. } e_k^{i,base} - e_{k,j}^{i,red} \ge 0 \\ \quad e_{k,j}^{i,red} \ge 0 \end{cases} \tag{4}$$

The consumers cannot provide an energy reduction $e_{k,j}^{i,red}$ higher than the reported consumption $e_k^{i,base}$, which they earlier transmitted considering the base price. Participation in these electricity auctions is generally categorized in atomic bids or combinatorial bids [53]. An atomic bid contains only information about a reward-reduction pair, whereas a combinatorial bid has multiple atomic bids. Different relationships can be defined in the combinatorial case using *AND*, *OR*, or *XOR* operators among atomic bids. These operators express the acceptance conditions of the participant. When offers are transmitted using the *AND* operator, the RA accepts all WDP results and executes all the associated reward-reduction pairs in the subsequent time slot. Conversely, when bids are sent using the *OR* operator, the RA can choose to implement any allocated winning bid. This paper considers that the RA transmits the offer set using the *XOR* operator, meaning that the SFMA can choose only one reward-reduction pair from the transmitted offers. According to [53], the RA bid set RA_{bidset}^i can be represented in a compact form for more than two atomic bids, as shown in (5).

$$RA_{bidset}^i = \left(\lambda_1^i, e_{k,1}^{i,red}\right) XOR \left(\lambda_2^i, e_{k,2}^{i,red}\right) XOR \ ... \ XOR \left(\lambda_M^i, e_{k,M}^{i,red}\right) \tag{5}$$

D. Assumptions

The SFM model considered here relies on the following assumptions.

1) The RA has signed a contract with the SFMA before presenting the bidding offers, which results in an obligation for the RA to execute the energy reduction determined in the auction results.

2) A reliable communication channel exists between the SFMA, the RAs, and the DSO.

3) Each time slot of the SFM is of a 5 min duration, and the SFMA transmits 10 discrete reward points. Previous studies have shown the convenience of using 10 reward points [54].

Without the first assumption, finding willing consumers for the SFM becomes time-consuming, particularly as the real-time event approaches. Disregarding the second assumption, which involves the lack of a reliable communication link between key agents, could undermine congestion reduction efforts through the SFM. Neglecting the third assumption about short time slots would force RAs to wait until the end of long-duration time slots if winning bids cause consumer discomfort.

III. WDP

In a combinatorial auction, the WDP involves auctioneer agents evaluating all combinations of atomic bids to identify the feasible set that satisfies allocation rules and checking the corresponding profit [18]. The winning bids in the bid set of each participant are those that maximize the profit within the feasible set. The WDP in the SFM depends on the transmitted bids from the RAs, which are discrete reward-reduction bid points. In each time slot of the SFM, the SFMA can receive an energy reduction request from the DSO to alleviate grid congestion. The request must be inside a feasible reduction region, as shown in Fig. 2 [55]. The red curve represents the maximum possible aggregated reduction, corresponding to the highest bids of each RA. Likewise, the blue curve represents the summation of the minimum energy reduction bids. Notably, the minimum energy reduction cannot be zero since all participating RAs must be assigned, and void bids are not allowed.

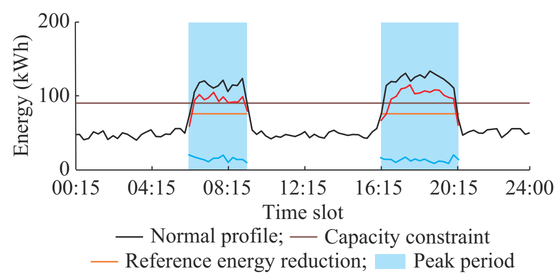

Fig. 2. Aggregated reduction boundaries of RA bids in each slot of SFM.

The SFMA is interested in maximizing its profit as the difference between the DSO payment and the RA rewards [56]. Thus, (6) represents the WDP for the SFM as a combinatorial optimization problem.

$$\begin{cases} \max\limits_{\lambda_j^i, \forall i \in I, \forall j \in J} \left(\xi_k \sum\limits_{i=1}^{N} \sum\limits_{j=1}^{M} b_j^i e_{k,j}^{i,red} \lambda_j^i - \sum\limits_{i=1}^{N} \sum\limits_{j=1}^{M} b_j^i e_{k,j}^{i,red} \lambda_j^i \right) \\ \text{s.t. } \sum\limits_{i=1}^{N} \sum\limits_{j=1}^{M} b_j^i e_{k,j}^{i,red} \lambda_j^i \le E_{ref}^{DSO} \\ \quad \sum\limits_{i=1}^{N} \min\limits_j e_{k,j}^{i,red} \le E_{ref}^{DSO} \le \sum\limits_{i=1}^{N} \max\limits_j e_{k,j}^{i,red} \\ \quad \sum\limits_{j=1}^{M} b_j^i = 1 \\ \quad b_j^i \in \{0, 1\} \end{cases} \tag{6}$$

where b_j^i is the binary variable; I is the set of N participating

RAs; and J is the set of M reward points.

The SFMA receives a compensation ξ_k per kWh reduced from DSO during the peak period up to the requested total energy reduction E_{ref}^{DSO} [46], [30]. b_j^i can only take value 0 or 1. This ensures that only one bid from each RA bid set in (5) is selected.

In literature, the exhaustive combinatorial search is the naive approach to solve the WDP with *XOR* bids in a single-sided auction. Meta-heuristic-based techniques can be used to diminish optimality. Besides, these techniques may suffer from variance in the outcomes due to stochastic-based search. The proposed approach in this paper overcomes these limitations by applying interior-point optimization with an approximate reward function to represent the RA bids. A similar previous approach, as discussed in [44], suggests an approximation for the cost function but does not consider constraints. The following subsections explain the complete WDP algorithm and the possible approximations of the *XOR* bid sets.

A. Approximation of XOR Bid Set

The RAs participating in the SFM must follow the auction rules. Accordingly, they shall present unique reduction bids against each reward point in a non-decreasing manner [54]. Once the SFMA receives the reduction bid set from each RA, the next step is to approximate the bids to a reward-based function f_{sel}^i. The considered functions here are linear, exponential, or fractional power raised due to the mentioned characteristics of the bid set. Minimizing the sum of squared residuals (SSR) tunes the function parameters, as presented in (7). The SFMA tests all three possible reward-based functions and chooses the one performing the lowest SSR for each RA.

$$\min_{\theta_i} SSR = \sum_{j=1}^{M} \left(e_{k,j}^{i,red} - f_{sel}^i\left(\lambda_j^i, \theta_i\right)\right)^2 \tag{7}$$

where θ_i is the learned parameter for functions.

The SFMA can provide different reward rates during the peak period according to the reduction offered [57]. Before each real-time energy exchange, the aggregator receives the ξ_k and the requested reduction. Thus, it completes all the information needed to solve the WDP. Interior-point solvers have been proven practical in solving comparable problems [58].

1) Linear Reward-based Function

In this case, the reward-based function approximates RA bids according to (8). As an illustrative example, Fig. 3(a) shows the energy reduction bids transmitted by one RA as discrete points. The blue dotted line depicts the corresponding linear regression curve constructed by the SFMA using (8).

$$f_{sel}^i\left(\lambda^i, \left[\alpha_i^{lin}, \beta_i^{lin}\right]\right) = \alpha_i^{lin} \lambda^i + \beta_i^{lin} \tag{8}$$

where α_i^{lin} and β_i^{lin} are the learned parameters for linear functions.

2) Exponential Reward-based Function

Due to the $e_{k,j}^{i,red}$ limits stated in (4), the linear residential bid set functions do not truly represent the bid points for some reduction sets.

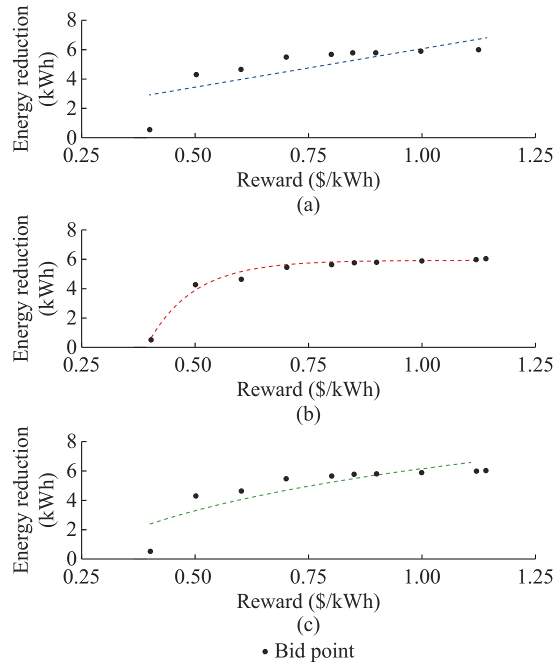

Fig. 3. Approximation of residential bid set. (a) Linear reward-based function. (b) Exponential reward-based function. (c) Fractional power raised reward-based function.

Thus, the SFMA may approximate the *XOR* bid set using an exponential reward-based function, as shown in (9). For example, the RA bid points presented in Fig. 3(b) get closer to an exponential regression. Again, minimizing SSR tunes the function parameters.

$$f_{sel}^i\left(\lambda^i, \left[\alpha_i^{exp}, \beta_i^{exp}, \gamma_i^{exp}\right]\right) = -\alpha_i^{exp} \mathrm{e}^{-\beta_i^{exp}\left(\lambda^i\right)} + \gamma_i^{exp} \tag{9}$$

where α_i^{exp}, β_i^{exp}, and γ_i^{exp} are the learned parameters for exponential function.

3) Fractional Power Raised Reward-based Function

For certain RA bid sets, the linear and exponential regressions do not indicate the best fit for f_{sel}^i. Accordingly, the SFMA can use a third option with a fractional power raised reward-based function, as presented in (10). Figure 3(c) shows an example of this case in green. Minimizing SSR finds the parameters α_i^{frc}, β_i^{frc}, and γ_i^{frc}.

$$f_{sel}^i\left(\lambda^i, \left[\alpha_i^{frc}, \beta_i^{frc}, \gamma_i^{frc}\right]\right) = \alpha_i^{frc}\left(\lambda^i\right)^{\beta_i^{frc}} + \gamma_i^{frc} \tag{10}$$

$$0 < \beta_i^{frc} < 1 \tag{11}$$

B. Winner Determination Algorithm

After approximating the discrete RA bid set (5) to a reward-based function (8), (9), or (10), the WDP presented in (6) can be reformulated as:

$$\begin{cases} \max\limits_{\lambda^i, \forall i \in I} \left(\xi_k \sum\limits_{i=1}^{N} f_{sel}^i\left(\lambda^i\right) - \sum\limits_{i=1}^{N} \lambda^i f_{sel}^i\left(\lambda^i\right) \right) \\ \text{s.t.} \quad \sum\limits_{i=1}^{N} f_{sel}^i\left(\lambda^i\right) \le E_{ref}^{DSO} \\ \quad\quad \sum\limits_{i=1}^{N} \min\limits_{j} e_{k,j}^{i,red} \le E_{ref}^{DSO} \le \sum\limits_{i=1}^{N} \max\limits_{j} e_{k,j}^{i,red} \end{cases} \tag{12}$$

Approximating the *XOR* bid set to f_{sel}^{i} of λ^{i} ensures the selection of a unique bid from each RA. Thus, $f_{sel}^{i}\left(\lambda^{i}\right) \approx e_{k,j}^{i,red}$ for each reward point in J transmitted to each RA in I. $\sum_{i=1}^{N}\lambda^{i}\left(f_{sel}^{i}\left(\lambda^{i}\right)\right)$ represents the payments from the SFMA to the RAs, which can take either an affine or convex form based on the function selected for each RA, as shown in *Step 2* of Algorithm 1. Once the SFMA has obtained the solution $\lambda^{i,*}$ to the problem presented in (12), the optimal value of $f_{sel}^{i,*}$ may not match any value in the discrete bid set of the RA_{bidset}^{i}, where $f_{sel}^{i,*}=f_{sel}^{i}\left(\lambda^{i,*}\right)$. Therefore, the SFMA computes the difference of $f_{sel}^{i,*}$ with all the bid points and identifies the index of the closest bid, as presented in (13). Algorithm 1 provides the complete procedure for finding the optimal bids in the CSA under the energy reduction constraint. For every time slot of SFM, the SFMA needs to update the approximated function (*Step 1* of Algorithm 1) based on the RA bid set for determining the winning bid.

$$RA_{index}^{i}=\underset{j\in J}{\arg\min}\left(\left|e_{k,1}^{i,red}-f_{sel}^{i,*}\right|,\left|e_{k,2}^{i,red}-f_{sel}^{i,*}\right|,...,\left|e_{k,M}^{i,red}-f_{sel}^{i,*}\right|\right) \tag{13}$$

where RA_{index}^{i} is the index of optimal bid.

Algorithm 1: winner determination based on energy reduction request from DSO

Input: reference energy reduction request from DSO and bid set from all participating RAs
Output: winning bid combination
Begin
for $i=1,2,...,N$ do
Step 1: approximate each RA bid set to f_{sel}^{i} using (8), (9), or (10)
Step 2: select f_{sel}^{i} for each RA that gives the lowest SSR
end
Step 3: solve the WDP (12)
Step 4: find the index of optimal bid for each RA using (13)
end
Step 5: announce the winning bid from RA_{bidset}^{i} (5) to RAs
Step 6: confirm the reduced energy allocation to DSO

Remark The literature proposes several solutions to non-convex problems in combinatorial auctions, including graph neural networks (GNNs), meta-heuristic-based techniques, mixed-integer programming (MIP), and branch and cut approaches. The integer condition in the WDP makes it non-convex and NP-hard [59]. To address this issue, [59] suggests using GNNs to understand the underlying probability distribution and learn the mapping of flexibility market bids of local energy to optimal solutions within a supervised learning framework. Reference [60] introduces a meta-heuristic genetic search for non-convex optimization problems, using randomized selection to improve the objective function. Without relying on gradient information, the solution can fall into local optima. Reference [61] presents a solution to the non-convex electricity day-ahead auction using a primal-dual framework, benefiting from parallel routines with state-of-the-art MILP solvers. Additionally, [62] proposes new valid inequalities for the branch and cut algorithm to solve the WDP, offering approaches to reduce problem size before solving.

IV. Simulation Results

This section elaborates on the performance of the proposed approach for the WDP under a given energy reduction request. The data used to tune the house thermal models of RAs correspond to actual energy demand and indoor temperature measures from houses in Quebec province, Canada, during 2018. The simulation runtime is measured on an Intel Core i7 (2.00 GHz) computer with 32 GB RAM. Due to computational resource limitations, the exhaustive combinatorial search approach is possible only for up to 8 houses. The DSO sends E_{ref}^{DSO} and the value of ξ_k for the peak period. These values constrain the SFMA profit. Notably, $\lambda_j^i<\xi_k$, $\forall j$, so the SFMA always has a profit margin. Table II shows the different energy reduction requests according to the number of RAs participating in the SFM.

TABLE II
Deferent Energy Reduction Requests

Number of RAs	E_{ref}^{DSO} (kWh)	ξ_k	Number of RAs	E_{ref}^{DSO} (kWh)	ξ_k
3	10.5	1.40	15	49.0	1.50
4	12.0	1.50	20	73.5	1.55
5	13.0	1.56	30	104.0	1.59
6	15.2	1.57	40	145.0	1.70
7	26.0	1.58	60	190.0	1.60
8	25.2	1.70	80	300.0	1.90
10	32.5	1.50	100	380.0	3.20

The parameters utilized in the simulation analysis for both the proposed approach and hybrid particle swarm optimization-genetic algorithm (HybridPSOGA) are shown in Table III [29], [63], [64].

TABLE III
Parameters Used in Simulation Analysis

Approach	Parameter	Description	Value	Ref.
Proposed approach	*max_nfev*	The maximum evaluation limit	10000	[64]
Hybrid-PSOGA	c_1, c_2	Learning coefficient	2.05, 2.05	[63], [29]
	w_{damp}	Weight damping ratio	0.9	
	mut_{fac}	Population mutation percentage	20	
	Population	Population size	100	
	Iteration	The maximum iteration	100	

The HybridPSOGA is an iterative approach that begins by generating a swarm of particles, and each assigned a random position index. Each particle adjusts its position index X_{iter}^{i} based on the distance to its best position $p_{best}^{x^i}$ and the global best position $g_{best}^{x^i}$. This adjustment is determined by the velocity concept v_{iter+1} [63]. The new position of each particle is updated by (14), where the velocity update is defined by (15).

$$X_{iter+1}^{i}=X_{iter}^{i}+v_{iter+1} \tag{14}$$

$$v_{iter+1}=w_{damp}v_{iter}+c_1r_1\left(p_{best}^{x^i}-X_{iter}^{i}\right)+c_2r_2\left(g_{best}^{x^i}-X_{iter}^{i}\right) \tag{15}$$

where r_1 and r_2 are the random variables with values ranging from 0 to 1. A genetic algorithm further refines the particles with the best positions, and a mutation factor mut_{fac} mutates their positions [29].

The reward points for energy demand reductions are the same for all participating RAs and are transmitted, as shown in Table IV.

TABLE IV
REWARD POINTS SENT TO RAs

Reward point	Value	Reward point	Value
λ_1^i	0.40	λ_6^i	0.85
λ_2^i	0.50	λ_7^i	0.90
λ_3^i	0.60	λ_8^i	1.00
λ_4^i	0.70	λ_9^i	1.12
λ_5^i	0.80	λ_{10}^i	1.14

A. Energy Reduction Results

The result of the proposed approach for clearing the flexibility market based on a reference energy reduction scenario is presented, as shown in Table V.

Other results of the approaches are presented for comparison: the exhaustive combinatorial search approach for CSA, the HybridPSOGA [65], the uniform reward auction (URA), and dynamic programming (DP).

The uniform price auction [66] has been modified into URA, and instead of transmitting the price point, the SFMA sends reward points to allow for more flexible bidding. Thus, the market is cleared by assigning the same reward to all participants. The HybridPSOGA is based on [29] and [67]. CSA can be observed as the naive approach because it evaluates all the possible combinations before announcing winners. However, it suffers from the curse of dimensionality when the number of RAs increases. Indeed, the CSA is unfeasible for SFM due to the short clearing time requirements. Finally, DP is also a searching algorithm, but it avoids evaluating all possibilities when possible. Consequently, DP is faster than CSA for the majority of scenarios.

The results show that CSA is the best approach since it gets closer to the reduction request. The HybridPSOGA provides an average μ close to the combinatorial results but has a variance σ. On the other hand, the proposed approach achieves an energy reduction consistently close to the demand reduction request from DSO. The worst-performing approach is URA since the achieved reductions are far from the DSO request. Certainly, choosing the same reward for all participants is an unfavorable strategy because it disregards the preferences and flexibility of individual RA.

In extension, Fig. 4 compares energy reduction achieved for the various approaches with up to 100 RAs participating. The CSA becomes unfeasible as the number of RAs grows. The purple shadow area represents the variance of HybridPSOGA.

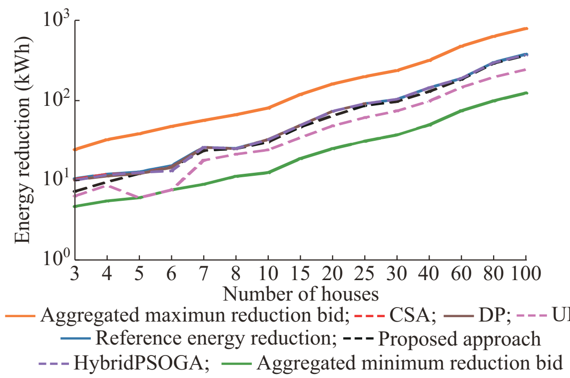

Fig. 4. Energy reduction under demand reduction request from DSO for up to 100 RAs.

B. Results of SFMA Profit

The SFMA profit using various approaches is compared in Table VI. The CSA achieves the highest possible profit by respecting constraints. The DP (the maximum profit) attains comparable profits. The HybridPSOGA also earns a profit but with some variance, as presented in Table VI. Since the energy reduction through the proposed approach is near the requested reference, it has an advantage over the URA and HybridPSOGA regarding profit level. The benefit of the proposed approach is more noticeable as the number of RAs increases. The *XOR* bid approximations impede achieving optimality but give an acceptable result compared with the CSA.

Figure 5 illustrates the evolution of SFMA profit with up to 100 RAs participating. The proposed approach consistently outperforms HybridPSOGA and URA. HybridPSOGA variance changes in the tested scenarios, managing to approach the optimal results occasionally. The profit results from URA are directly proportional to the difference between its achieved energy reduction and the DSO request.

TABLE V
ENERGY REDUCTION UNDER E_{ref}^{DSO} RESTRICTION

Number of RAs	E_{ref}^{DSO}	Energy reduction (kWh)							
		The maximum bid boundary	The minimum bid boundary	Proposed approach	CSA	DP	μ (HybridPSOGA)	σ (HybridPSOGA)	URA
3	10.5	24.380	4.700	7.270	10.360	10.170	10.100	0.003	6.35
4	12.0	32.400	5.500	9.550	11.960	11.400	11.660	0.110	8.66
5	12.8	38.420	6.030	12.160	12.300	12.180	12.430	0.200	6.03
6	15.2	47.440	7.570	14.590	14.730	14.610	14.300	2.160	7.57
7	26.0	56.460	8.900	23.670	25.880	25.880	25.970	0.040	17.82
8	25.2	66.480	11.230	24.890	25.010	24.890	24.730	0.440	21.25

TABLE VI
SFMA PROFIT UNDER E_{ref}^{DSO} RESTRICTION SHOWN IN TABLE V

Number of RAs	SFMA profit ($)					
	Proposed approach	CSA	DP	μ (HybridPSOGA)	σ (HybridPSOGA)	URA
3	6.12	8.78	8.29	10.10	0.003	5.72
4	9.31	10.90	10.41	11.66	0.110	8.66
5	12.93	13.24	12.98	12.43	0.200	7.00
6	15.66	15.96	15.71	14.30	2.160	8.85
7	23.90	25.83	25.83	25.97	0.040	19.26
8	28.63	29.29	28.63	24.73	0.440	25.50

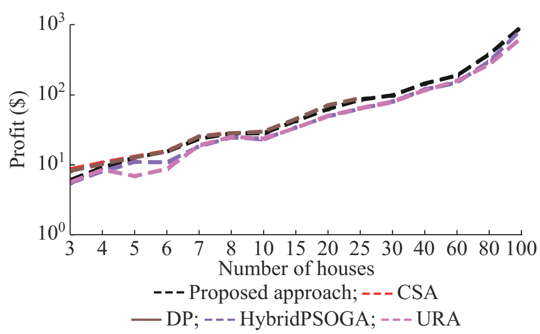

Fig. 5. SFMA profit comparison under demand reduction request from DSO for up to 100 RAs.

The energy reduction achieved through HybridPSOGA is near the reference request from the DSO. Thus, its higher profit variance is partly due to the existence of bid combinations with the same energy reduction at different costs.

C. Profit Loss Comparison

The efficiency of the proposed approach is assessed through various simulations involving 7 consumers randomly submitting bid set (5) in the flexible auction market. In each scenario, E_{ref}^{DSO} and ξ_k are set to be 25 kWh and \$1.58, respectively. In Fig. 6(a), the CSA consistently respects the E_{ref}^{DSO}. In Fig. 6(b), it is demonstrated that solving (12) in the continuous domain also maintains compliance with the flexibility purchase constraint. Figure 6(c) shows an adherence to E_{ref}^{DSO} when mapping the continuous solution obtained through (12) to a discrete bid set (5) using Algorithm 1, as shown in teal green. Approximately 48% of the values in overall random scenarios exceed E_{ref}^{DSO}. The extent of exceeding values depends on the spacing of the bids in the RA bid set (5). The increased dispersion of bids in the bid set (5) leads to a higher loss in adhering to E_{ref}^{DSO}. In contrast, when RAs submit closely spaced bids, it will lead to an aggregated reduction, facilitating the compliance with E_{ref}^{DSO}. The adjusted Algorithm 1 is depicted in grey in Fig. 6(c). In adjusted Algorithm 1, the SFMA ensures adherence to constraint by adjusting E_{ref}^{DSO} as $E_{ref,new}^{DSO}=E_{ref}^{DSO}-\kappa$, where κ is continuously adjusted to ensure compliance when mapping from continuous to discrete bid set (5). The HybridPSOGA, depicted in purple in Fig. 6(c), satisfies E_{ref}^{DSO} in all scenarios. Note that UL and LL denote the upper and lower boundaries, respectively.

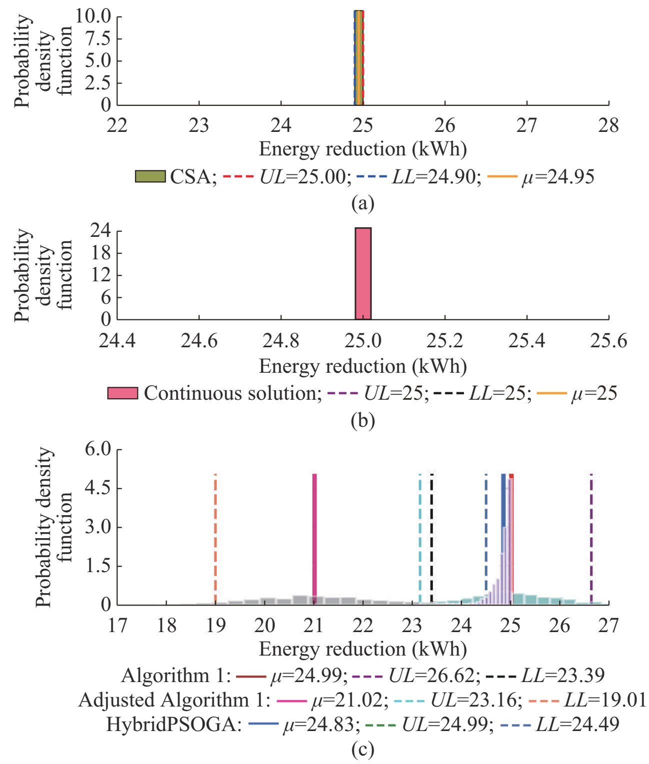

Fig. 6. Energy reduction comparison of all approaches among seven RAs with randomized bid set and fixed demand reduction request from DSO. (a) With CSA. (b) With continuous solution. (c) With three approaches.

The profit obtained using the CSA for the WDP is illustrated in olive green in Fig. 7. This approach serves as the benchmark for solving the WDP. The profit earned from the utilization of Algorithm 1 is depicted in teal green in Fig. 7. The profit obtained by adjusted Algorithm 1 is shown in grey in Fig. 7.

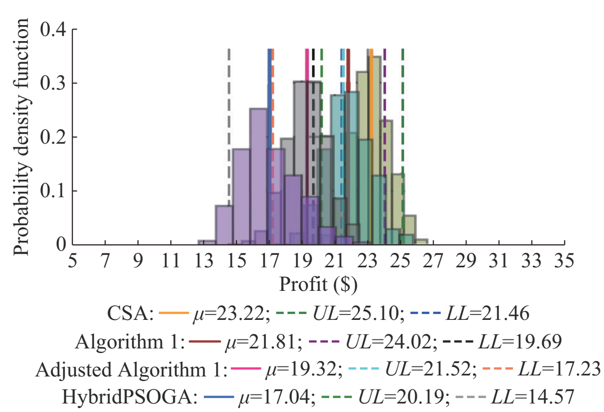

Fig. 7. Profit earned comparison of all methods among seven RAs with randomized bid set and fixed demand reduction request from DSO.

The adjusted Algorithm 1 respects the constraint, which results in a lower profit mean compared with Algorithm 1. The profit earned through HybridPSOGA is represented in purple.

The analysis focuses on evaluating the profit loss in percentage compared with the solution provided by CSA. The histogram in Fig. 8 illustrates the profit loss results through the Algorithm 1 (teal green), adjusted Algorithm 1 (gray), and HybridPSOGA (purple). The proposed approach helps the SFMA perform better than the adjusted Algorithm 1 and the HybridPSOGA. This is particularly important in the context of short-duration SFM because it gives a better trade-off between market clearing speed and financial efficiency.

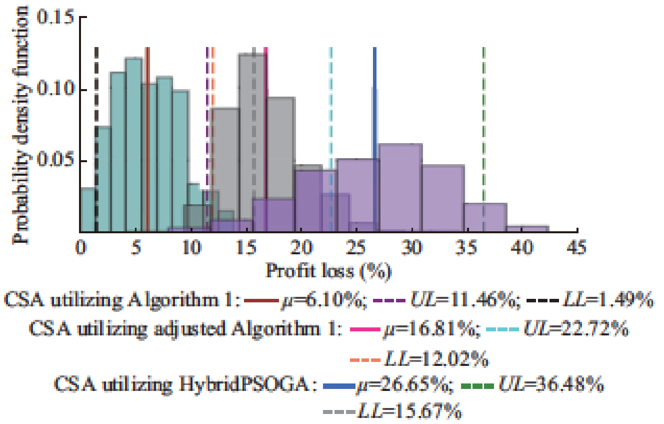

Fig. 8. Comparison of profit loss in percentage for three approaches with respect to CSA in Fig. 7.

D. Computational Complexity

Reducing computational complexity is essential for SFMA, particularly when managing a substantial number of consumers. For instance, as presented in [68], aggregators often interact with more than 1000 residential customers. In such scenarios, the market clearing time in the SFM is essential for a continuous and reliable operation. The SFMA must quickly choose the optimal bids to maximize its profit while responding to DSO requests. Figure 9 shows the computational time comparison of all approaches. The advantage of the proposed approach compared with CSA and DP is evident as those approaches suffer from the curse of dimensionality with large sets of RAs. Considering M as the 10 bids from each RA, the CSA has a time complexity of $\mathcal{O}\left(M^N\right)$, while the interior point solver used in the proposed approach has $\mathcal{O}\left(\sqrt{N}\ln\left(N/\omega\right)\right)$ for an ω accuracy [69]. The proposed approach is also faster in computational speed than HybridPSOGA in all considered scenarios.

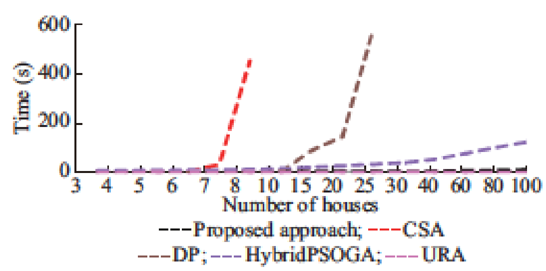

Fig. 9. Comparison of computational time all approaches.

In Fig. 10, the decision-making using various approaches is illustrated over the entire search space when 6 RAs participate with SFMA in the presence of E_{ref}^{DSO}.

In Fig. 10, all possible profit outcomes from CSA are depicted as grey dots, with the optimal bid that maximizes profit highlighted in red. URA serves as a baseline approach without combination formation. Each aggregation is evaluated against discrete incentive points, denoted by pink asterisks, with the maximum profit respecting constraints marked by a large pink asterisk. URA expedites market clearing but yields only 10 possible profits, resulting in significant profit loss compared with CSA. DP starts with a URA solution, and its computational costs escalate beyond 25 houses. The HybridPSOGA respects constraints but exhibits varying profit levels due to bid combination variability.

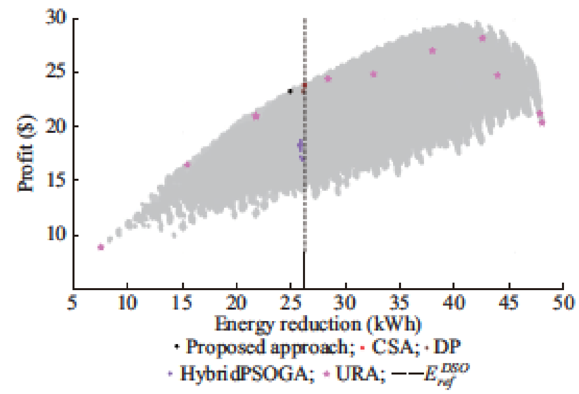

Fig. 10. Evidence of various decision-making approaches over entire search space.

The proposed approach approximates RA bids to a reward-based function, closely aligning with the global combinatorial approach and ensuring consistent solutions across iterations.

E. Response Time

The response time is calculated using a co-simulation platform employing Raspberry Pi 4B+ devices as RA, as shown in Fig. 11. The SFMA interacts with RAs twice before running Algorithm 1 to solve the WDP. Initially, RAs run thread 1 to optimize base energy demand reporting, with a maximum execution time of approximately 17.67 s per RA [70]. The aggregator then waits for all RA responses, taking a maximum of 21.69 s for up to 100 houses, totalling 39.36 s for the first interaction.

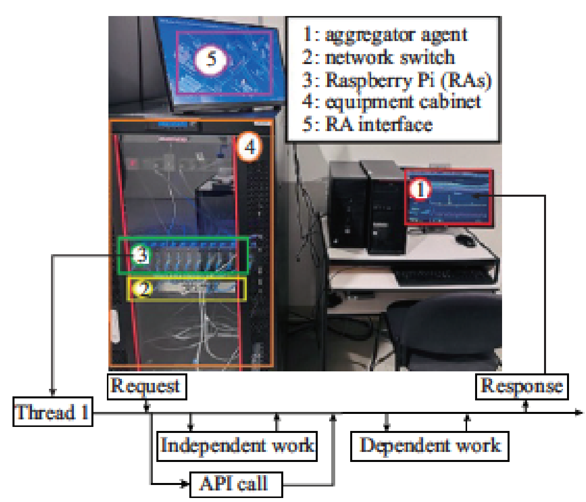

Fig. 11. Co-simulation platform for interaction of RA and SFMA.

Subsequently, the aggregator sends incentive point sets to all RAs, followed by RA optimization processes to determine energy reductions. The maximum execution and communication time remains similar, summing to another 39.36 s. The aggregator then runs Algorithm 1 to solve the WDP, allowing flexibility for the DSO to manage congestion in the SFM. The approximate time for interactions and Algorithm 1 is 80.24 s for up to 100 RAs, consuming about 26.75% of the 5 min SFM time slot. The remaining time leaves room for real-time market variability, delays, or increased RA numbers.

V. Conclusion

SFMs permit DSOs to alleviate system congestion by facilitating demand-side management. These markets comprise single-sided auctions where an SFMA intermediates to manage a group of customers. The success of SFMA depends on fast clearing mechanisms that provide requested energy reductions at competitive prices. This paper presents a clearing mechanism that reduces computational complexity compared with state-of-the-art approaches. The effectiveness of the proposed approach in finding an acceptable profit of SFMA is validated, even for a large set of customers. It leverages reward-based function approximations and interior-point solvers. Reducing computational complexity is vital for implementing SFMs, so exploiting the presented approximations for discrete bid sets is advisable. Furthermore, the developed formulation of the WDP eases the use of commercial solvers. The achieved energy reductions show the feasibility of the proposed approach to meet DSO demands.

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