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Successive Column-wise Matrix Inversion Update for Large Scale Massive MIMO Reciprocity Calibration

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***Abstract*—In this paper we consider an efficient method to resolve the underlying large matrix inversion problem inherent in the antennas' mutual coupling based reciprocity calibration. Such calibration enables the downlink pre-coding using the uplink channel estimates in a time-division-duplex (TDD) massive MIMO systems. Based on least squares estimators, a large matrix inversion is required. Herein, we derive an efficient method based on successively updating a matrix inverse by exploiting the Gram matrix structure. The simulation results reveal that our proposed method performs as well as the direct matrix inversion (based on Cholesky decomposition) whereas the approximation techniques based on Gauss Seidel (GS) and Neumann series expansions (NSE) require a large number of iterations. The proposed method is computationally efficient and lend itself for an efficient parallel and pipelined architecture implementation.**

***Index Terms*—Massive MIMO, reciprocity calibration, Gauss Seidel, Neumann series expansions, Cholesky decomposition, Least square, matrix inversion lemma, precoding.**

I. Introduction

Massive MIMO (also referred to as "Large-Scale MIMO") has been selected as one of the main disruptive technologies for 5G [1][2]. Massive MIMO is a form of multiuser MIMO wherein the number of serving antennas at the base transceiver station (BS) is much larger than the number of user terminals (UTs) served within each radio resource element [3]-[4]. In its canonical form, a large-scale MIMO system operates in time division duplex (TDD) mode, where the downlink and uplink transmissions are operating in the same frequency resource but are separated in time. Given the large number of antennas, reliance on TDD channel reciprocity is essential [5]. Basically, massive MIMO systems exploit this reciprocity to estimate the channel responses on the uplink and then use the acquired channel state information (CSI) for both uplink receive combining/detection and downlink transmit precoding/beamforming of the users' payload data. CSI is usually acquired by transmitting predefined pilot signals and estimating the channel coefficients from the received signals [6]-[7].

Unfortunately, being part of the end-to-end downlink and uplink channels, the different radio frequency transceiver chains make the overall channel not reciprocal. Therefore, a reciprocity calibration mechanism is required to estimate and compensate for such hardware impairments [8]. The authors in [8] addressed the non-reciprocity impact and the possible compensation solutions. It has also been reported that the downlink precoding performance degrades due to inaccurate reciprocity calibration [8]-[9]. The reciprocity calibration issue has been generally discussed in [10] whereas a novel calibration approach was proposed in [11]. Implemented in a real testbed, one of the base station's (BS) antennas is used as a reference element to which every other BS's antenna successively transmit and receive a pilot signal. However, this scheme is sensitive to the position of the reference antenna. Access points calibration in a distributed massive MIMO system has been proposed in [5] and [12] as a generalization of the method in [11]. The authors in [9] extended the framework where several least squares (LS) based estimators are proposed. The calibration is entirely performed at the base station (BS) by exploiting the antenna coupling to sound the BS antennas one-by-one. On the other hand, the authors in [13] proposed an expectation–maximization (EM) algorithm that processes the measured channels in order to estimate the calibration coefficients. Like LS based methods [12], the EM algorithm is also suitable for both collocated and distributed Massive MIMO BS.

Based on LS problem formulation, the closed form solution involves a large matrix inversion of $(M-1)\times(M-1)$, where M is the number of the BS antenna elements. Even if it has been argued that the calibration coefficients vary slowly with the time, it shall be stressed that such calibration shall be performed very often, especially in a distributed Massive MIMO setup where different low quality reference clocks can be used. Therefore, an efficient large matrix inversion technique is required.

Several implicit and explicit matrix inversion techniques are used for Massive MIMO detection and precoding [14]-[15]. Neumann series expansion and Gauss Seidel techniques are often used as computationally efficient techniques to solve a fairly large linear system of equations [14]-[15]. The latter has shown to converge with a small number of iterations. Unfortunately, these methods require a large number of iterations for large dimension matrix inversion inherent in the LS problem (see simulation results discussion section). In this paper we propose a novel successive matrix inversion update method by applying *successively* the matrix inversion lemma *column-by-column* to invert a matrix of the form $\mathbf{A}^H\mathbf{A}$, where $\mathbf{A}$ is a $M\times(M-1)$ matrix. The performance, in terms of the mean square error (MSE), and the computational complexity evaluation, in terms of the number of complex multiplications, are discussed in the performance analysis section.

The paper is organized as follows: Section II summaries the LS problem formulation. Section III details the proposed method whereas section IV shows and discusses some simulation results. Finally, a conclusion is drawn.

II. Least Square Problem Formulation for Reciprocity Calibration

A. System model for coupled BS-to-BS Signals

This section summaries the system model for BS-to-BS signals. It is not intended to provide an in-depth discussion on the model which is thoroughly addressed in [9] and [11]. The calibration coefficients b_m , where $m = 1,\cdots,M$, are estimated by sounding every antenna element one at a time. Each l antenna element transmits a pilot signal $s_p = +1$, while every m antenna element, among other $M-1$, receives a signal which we denote $y_{m,l}$. Gathered in pairs, the received signals are written as [9]

$$\begin{bmatrix} y_{l,m} \\ y_{m,l} \end{bmatrix} = \alpha_{l,m} \begin{bmatrix} b_l \\ b_m \end{bmatrix} + \begin{bmatrix} n_{l,m} \\ n_{m,l} \end{bmatrix} \tag{1}$$

where $\alpha_{l,m} = t_l^B t_m^B \tilde{h}_{l,m} = t_m^B t_l^B \tilde{h}_{m,l}$, t_m^B and t_l^B are complex coefficients (gain and phase) introduced by the m-th and the l-th RF chains of the BS, and $\tilde{h}_{l,m} = \tilde{h}_{m,l}$ is the reciprocal propagation channel. $\begin{bmatrix} n_{l,m} & n_{m,l} \end{bmatrix}^T$ is a vector whose components are independent zero-mean circularly symmetric complex Gaussian distributed random noises with variance N_0 per component.

Based on real world measurements, an empirical model for $\tilde{h}_{l,m}$ was proposed as $\tilde{h}_{l,m} = \beta_{l,m} \exp\left(j\phi_{l,m}\right) + w_{l,m}$, where $\beta_{l,m} = 0.03 d_{l,m}^{-3.7}$ with the distance $d_{l,m}$ between the m-th and the l-th antennas is in multiple of half the wavelength [9].

The channel phase $\phi_{l,m}$ is modeled as a uniformly distributed variable within $[0,2\pi]$ while the Rayleigh component $w_{l,m}$ is assumed to be a complex Gaussian circularly symmetric with variance N_w .

B. Least squares estimators

The direct path method in [11] estimates the calibration coefficients $\mathbf{b} = \begin{bmatrix} b_0, & b_1, \ldots, & b_{M-1} \end{bmatrix}^T$ using the received signals $y_{0,m}$ and $y_{m,0}$ where the direct path LS estimate is

$$\hat{b}_m = y_{0,m} / y_{m,0} \tag{2}$$

On the other hand the generalized LS [9] uses the full $M \times (M-1)$ received signals to minimize the following cost function

TABLE 1. The Proposed algorithm for Successive Column-wise Matrix Inversion Update (SCwMIU)

INPUT: $\mathbf{X} = \mathbf{A}_1$		
Precompute the scalar inverse as a starting point:		
$\mathbf{B}_1 = \left(\mathbf{X}_{1:1}^H \mathbf{X}_{1:1}\right)^{-1}$		// $\mathbf{B}_1$ is an 1×1 matrix (scalar)
FOR $m = 2$ **to** $M-1$ **DO:**		
1.	$\mathbf{z} = \mathbf{X}_{m:m}$	// $\mathbf{X}_{m:m}$ is the $m-$th column of $\mathbf{X} = \mathbf{A}_1$
2.	$\mathbf{y}_1 = \mathbf{X}_{1:m-1}^H \mathbf{z}$	// $\mathbf{X}_{1:m-1}$ is a matrix of columns 1 to $m-1$ of $\mathbf{X} = \mathbf{A}_1$
3.	$\mathbf{y}_2 = \mathbf{B}_{m-1} \mathbf{y}_1$	
4.	$c = 1 / \left(\mathbf{z}^H \mathbf{z} - \mathbf{y}_1^H \mathbf{y}_2\right)$	
5.	$\mathbf{y}_3 = c\, \mathbf{y}_2$	
6.	$\mathbf{\Gamma} = \mathbf{B}_{m-1} + c\, \mathbf{y}_2 \mathbf{y}_2^H$	
7.	$\mathbf{B}_m = \begin{bmatrix} \mathbf{\Gamma} & -\mathbf{y}_3 \\ -\mathbf{y}_3^H & c \end{bmatrix}$	// $\mathbf{B}_m$ is a growing $m \times m$ matrix
END DO		
OUTPUT: $\mathbf{B}_{M-1} = \left(\mathbf{A}_1^H \mathbf{A}_1\right)^{-1}$		

$$J(\mathbf{b}) = \sum_{m,l \neq m} \left\| b_m y_{m,l} - b_l y_{l,m} \right\|^2 \tag{3}$$

which admits this closed form solution

$$\hat{\mathbf{b}} = \begin{bmatrix} \hat{b}_1, & \hat{b}_2, \ldots, & \hat{b}_{M-1} \end{bmatrix}^T = -\left(\mathbf{A}_1^H \mathbf{A}_1\right)^{-1} \mathbf{A}_1^H \mathbf{a} \tag{4}$$

where $\mathbf{a}$ is the first column of $\mathbf{A} \triangleq \begin{bmatrix} \mathbf{a} & \mathbf{A}_1 \end{bmatrix}$ with $b_0 = 1$ to avoid the trivial solution $\mathbf{b} = \mathbf{0}$. The elements of the matrix $\mathbf{A}$ are defined as

$$\mathbf{A}_{m,l} = \begin{cases} \sum_{l=0}^{M-1} \left| y_{m,l} \right|^2, & m = l \\ -y_{m,l}^* y_{l,m}, & m \neq l \end{cases} \tag{5}$$

Other LS variants such as weighted LS are well discussed in [9].

C. Total least squares estimator

Equation (4) can be written as $\mathbf{A}_1 \hat{\mathbf{b}} = -\mathbf{a}$. The LS solution is optimal if all errors are confined into $\mathbf{a}$ and they have Gaussian distribution. The elements in matrix $\mathbf{A}_1$ are assumed to be free from errors [16]. Unfortunately, these conditions are not satisfied for the reciprocity calibration problem. The total least squares (TLS) has been devised as a more global and reliable method when both $\mathbf{A}_1$ and $\mathbf{a}$ are based on noisy measurements. The TLS solution is based on the right singular vector $\mathbf{v} = \begin{bmatrix} v_{1,M}, & v_{2,M}, \ldots, & v_{M,M} \end{bmatrix}^T$ corresponding to the smallest singular value of the extended matrix $\begin{bmatrix} \mathbf{A}_1 & \mathbf{a} \end{bmatrix}$ wherein the TLS estimate is then given as

$$\hat{\mathbf{b}}_{TLS} = \frac{1}{v_{1,M}} \mathbf{v} \tag{6}$$

The normalization by $v_{1,M}$ in (6) is performed to set $\hat{b}_{TLS,0} = 1$. One shall expect that the generalized LS and the TLS to perform equally well at high SNR as the errors in $\mathbf{A}_1$ and $\mathbf{a}$ are mainly due to the noise vector $\begin{bmatrix} n_{l,m} & n_{m,l} \end{bmatrix}^T$.

III. Successive Column-wise Matrix Inversion Update

Notice that equation (4) involves a large $(M-1)\times(M-1)$ $\mathbf{A}_1^H\mathbf{A}_1$ matrix inversion. Several implicit and explicit matrix inversion approximation techniques can be used. One can resort, for instance, to Neumann series expansion (NSE) or Gauss Seidel (GS) techniques that have been successfully used to resolve the zero-forcing (ZF) and minimum mean square error (MMSE) based massive MIMO detection problem where a direct application of Cholesky or QR decomposition is computationally intensive. Unfortunately, as it is shown, through simulations in this paper, such methods will require a large number of iterations to converge in the context of solving the LS problem (4).

We therefore propose to apply the matrix inversion update lemma when a new column is added [17]. Assume we have the inverse of a $(M-2)\times(M-2)$ matrix $\mathbf{B}_{M-2}^{-1} = \mathbf{A}_{1:M-2}^H\mathbf{A}_{1:M-2}$ (note that we have adopted the Matlab index notation $1:M-2$ in $\mathbf{A}_{1:M-2}$ to designate all columns 1 to $M-2$ of $\mathbf{A}_1$). Therefore, the inverse of a $(M-1)\times(M-1)$ matrix $\mathbf{B}_{M-1}^{-1} = \mathbf{A}_{1:M-1}^H\mathbf{A}_{1:M-1}$ can be computed as follow

$$\begin{aligned}
\mathbf{B}_{M-1} &= \left(\mathbf{A}_{1:M-1}^H\mathbf{A}_{1:M-1}\right)^{-1} \\
&= \left(\begin{bmatrix} \mathbf{A}_{1:M-2}^H \\ \mathbf{A}_{M-1:M-1}^H \end{bmatrix}\begin{bmatrix} \mathbf{A}_{1:M-2} & \mathbf{A}_{M-1:M-1} \end{bmatrix}\right)^{-1} \\
&= \begin{bmatrix} \mathbf{A}_{1:M-2}^H\mathbf{A}_{1:M-2} & \mathbf{A}_{1:M-2}^H\mathbf{A}_{M-1:M-1} \\ \mathbf{A}_{M-1:M-1}^H\mathbf{A}_{1:M-2}^H & \mathbf{A}_{M-1:M-1}^H\mathbf{A}_{M-1:M-1} \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \mathbf{\Gamma} & -c\mathbf{B}_{M-2}^H\mathbf{A}_{1:M-2}^H\mathbf{A}_{M-1:M-1} \\ -c\mathbf{A}_{M-1:M-1}^H\mathbf{A}_{1:M-2}\mathbf{B}_{M-2}^H & c \end{bmatrix}
\end{aligned} \quad (7)$$

where

$$c = \frac{1}{\mathbf{A}_{M-1:M-1}^H\mathbf{A}_{M-1:M-1} - \mathbf{A}_{M-1:M-1}^H\mathbf{A}_{1:M-2}^H\mathbf{B}_{M-2}\mathbf{A}_{1:M-2}^H\mathbf{A}_{M-1:M-1}}$$ and

$\mathbf{\Gamma} = \mathbf{B}_{M-2} + c\mathbf{B}_{M-2}\mathbf{A}_{1:M-2}^H\mathbf{A}_{M-1:M-1}\mathbf{A}_{M-1:M-1}^H\mathbf{A}_{1:M-2}\mathbf{B}_{M-2}^H$.

Applying equation (7) *successively* from the second column all the way to the last column $M-1$, the algorithm, dubbed successive column-wise matrix inversion update (SCwMIU), is outlined in Table 1.

GS technique estimates the calibration coefficients iteratively using [15]

$$(\mathbf{D}+\mathbf{L})\hat{\mathbf{b}}^{(l)} = -\mathbf{A}_1^H\mathbf{a} - \mathbf{L}^H\hat{\mathbf{b}}^{(l-1)}, \quad l = 1,\ldots,L_{MAX} \quad (8)$$

where $\mathbf{D}$ and $\mathbf{L}$ are, respectively, the diagonal and the lower triangular part of $\mathbf{A}_1^H\mathbf{A}_1$. L_{MAX} is the maximum number of iterations. The calibration coefficients $\hat{\mathbf{b}}^{(0)}$ are initialized with the estimates from the direct LS method (2).

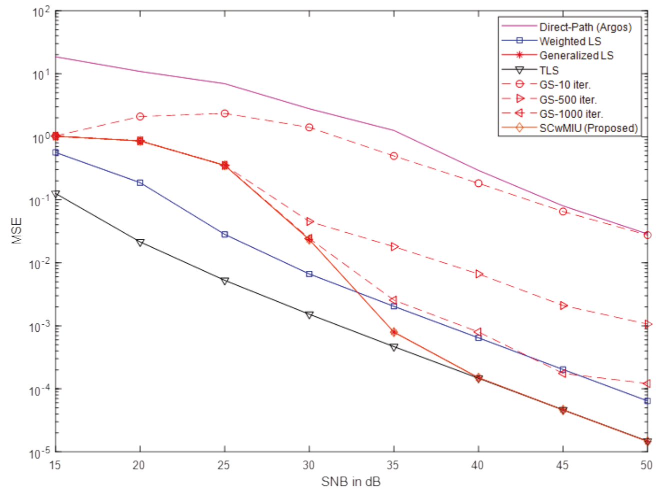

Fig. 1. Average mean square error (MSE) of the calibration coefficients versus the calibration SNR.

IV. Performance Analysis

A. Simulation set up and parameters

The simulation set up is based on similar parameters as in [9] for a 5x20 planar patch array, with the variance N_w of the channel Rayleigh component of -50dB. The center antenna (antenna element 49) is set as the reference element. The modeling of the RF chain statistics is based on [12] where the transmitter and the receiver factors t_m^B and r_m^B are assumed to have uniformly distributed phase within $[-\pi,\pi]$ and uniformly distributed magnitude within $[1-\delta, 1+\delta]$ where δ is selected in such a way that

$$\sqrt{E\left(\left(\left|t_m^B\right|-1\right)^2\right)} = \sqrt{E\left(\left(\left|r_m^B\right|-1\right)^2\right)} = 0.1 \quad (9)$$

where $E(\bullet)$ is the expectation operator.

The calibration SNR, in the simulation results (c.f. figure 1), is the ratio of the average received signal power and the total noise power.

B. Simulation results and discussion

Figure 1 depicts the mean square error (MSE) as a function of the calibration SNR. As expected the TLS outperforms the generalized and weighted LS methods at low calibration SNR. This is due to the fact that both $\mathbf{A}_1$ and $\mathbf{a}$ are made of noisy received pilot signals. The generalized LS performances equally well at high calibration SNR as the noise in both $\mathbf{A}_1$ and $\mathbf{a}$ is vanishing.

Of particular interest is the proposed SCwMIU method which does not suffer from any performance degradation compared to the generalized LS (based on direct matrix inversion). The proposed method can be applied to the weighted LS problem to improve its performance at low calibration SNR as well. Figure

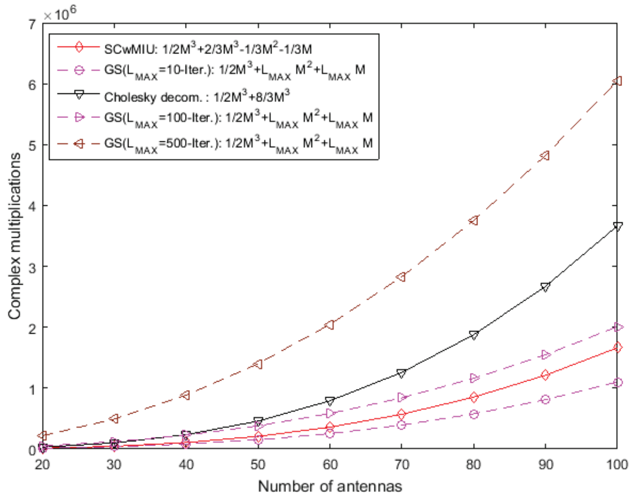

Fig. 2. Computational complexity of the proposed SCwMIU, GS and Cholesky decomposition based LS. Note that the term $1/2M^3$ is the contribution of the common operations in computing $\mathbf{A}_1^H\mathbf{A}_1$.

1. reveals that GS requires a large number of iterations to converge and so does (not shown here for the sake of clarity) NSE technique. Figure 2. depicts the computational complexity in terms of the number of complex multiplications as a function of the number of antenna and the maximum number of iterations for GS. The reference generalized LS is implemented using Cholesky decomposition. The results reveal that the proposed SCwMIU method is by far computationally efficient in such large scale matrix inversion issue inherent in the LS problem for reciprocity calibration in massive MIMO systems. At the expense of reduced performance, GS with less than 100 iterations can be used in the event maximum ratio transmitter precoding is used.

It is worth mentioning that the calibration errors have negative effect on the performance of the linear precoding techniques such as ZF. The effect on maximum ratio transmitter (MRT) and ZF precoding is well discussed in [8] and [9]. At an operating SNR of 20dB, it has been reported that significant loss, of more than 20%, in the sum rate is expected, for calibration SNR less than 35dB.

V. Conclusion

A massive MIMO system operates in TDD mode to justify the use of a low pilot overhead for channel estimation, while exploiting the channel reciprocity, so that the same estimated uplink channel state information is reused for the downlink precoding, within the same channel coherent time. Unfortunately, the radio frequency transmitters and receivers in the BS introduce random complex coefficients that alter such reciprocity. A reciprocity calibration mechanism is therefore required to achieve the promising high spectral and energy efficiencies. In this paper we have closely looked at the LS-based reciprocity calibration problem from the computation efficiency perspective to tackle the inherent large matrix inversion problem. Applying iterative techniques such GS, which have proven to be computationally efficient than direct matrix inversion methods (e.g. Cholesky decomposition) in the detection/precoding problem, requires a large number of iterations. We have proposed an efficient low computational complexity technique based on successively updating the matrix inverse which is originally expressed as a Gram matrix (i.e. a product of one matrix with its conjugate transpose). In fact, the prosed method computes an exact matrix inverse which does not show any performance degradation. The method is transparent in the sense that it can be generalized to weighted least square as well.

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