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WINDOWING COMPENSATION IN FOURIER BASED SURROGATE ANALYSIS

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ABSTRACT

This paper shows how adding a second step of windowing after each phase randomization can reduce the False Rejection Rate in Fourier based Surrogate Analysis. Windowing techniques improve the resolution of the Power Spectrum estimation by reducing the sampling gap caused by the periodic extension of the Fourier Series. However, it adds a time domain non-stationarity which affects the Surrogate Analysis. This effect is particularly problematic for short low-pass signals. Applying the same window to the surrogate data allows having the same non-stationarity. The method is tested on order 1 autoregressive process null hypothesis by Monte Carlo simulations. Previous methods were not able to yield good performances for left-sided and right-sided tests at the same time, even less with bilateral tests. It is shown that the new method is conservative for unilateral tests as well as bilateral tests.

***Index Terms*—**LTV, Surrogate Analysis, Surrogate Data, Nonlinear Analysis, Windowing

1. INTRODUCTION

The Surrogate Analysis (SA) is a hypothesis test aimed at assessing the nonlinear nature of a signal [1]. It has been applied to a wide variety of domain [2], notably in the study of brain activity [3, 4]. More recently, it has been used as a feature for Low-Back-Pain diagnostic with electromyogram (EMG) sensors [5] and to study the non-randomness of the phase spectrum in the Fourier domain [6]. It has also been extended for distinguishing between nonlinear and non-stationary data [7].

Multiple versions of the SA have been developed but can generally be categorized in two groups: Fourier based and ARMA process based. Both approaches were proposed in [1]. By far, the Fourier approach has been the most popular [8], probably for its simplicity of implementation since it does not require any model selection step. However, the Fourier based SA is well known for its sensitivity to signal artifacts. An important example of such artefacts is the impact of limited number of data. One solution to this artefact is the method matching ends. It was first proposed in [1] by recommending tailoring the data length in order to have similar first and last data point values. Later, it was analysed in [9] that matching the ends gave reasonable performances for reducing the effect of the "periodicity mismatch." (difference between the first and last data point) when the number of data available is high. Another solution is to apply windowing techniques [10] which have the advantage of not requiring a variable data length or initial point. This makes it easier to implement or to compare results between different time series, especially when the data length is small. Also, nonlinear methods can be biased by the data length [11]. Windowing techniques [10] improve the resolution of the Power Spectrum estimation needed for the generation of surrogate series in Fourier based SA by reducing the sampling gap caused by the periodic extension of the Fourier Series. In [1, 12], the windowing is applied in the generation of the surrogate's data. However, these surrogates are compared to the *unwindowed* original series. This leads to a spreading in the power spectrum of the surrogate signal which is not present in the original signal, producing a bias. The results showed the trade-off between reducing the sampling gap and avoiding the frequency spreading. Indeed, it indicated that using the windowing techniques is useful up to a certain data length. Above, the windowing worsens the bias. Moreover, even in the best cases, it remained largely optimistic, yielding between 10 and 25% of False Rejection Rate (FRR) when a 5% rate would be expected. Nonetheless, it is clear that the application of windowing techniques is important: without it, the FRR were shown to be over 30%.

A different approach would be to compare the surrogate to the windowed version of the original signal. However, the windowing process adds time domain non-stationarity. The windowing of a stationary ARMA process makes its variance changing from sample to sample. It has been shown that the SA may be very sensitive to non-stationarity [7, 13]. This effect has even been exploited notably in [14]. The non-stationarity caused by the windowing in the original data will not be present in the surrogate series. Clearly, the windowing adds a bias. The biases in SA make either the left or right-sided test over-optimistic. Also, it makes the bilateral tests always over-optimistic. The bilateral tests are important when the type of nonlinearity that might be present is unknown.

This paper presents a method for compensating the non-stationarity caused by the windowing in surrogate analysis. The method consists of keeping the window on the original series and applying the same window on every surrogate data series. This adds the same non-stationarity to the surrogate series as in the original series. The analysis is conducted by Monte Carlo simulations on an order 1 autoregressive process

(AR(1)) as in [12], but considering the unilateral and bilateral tests. Although the added windowing replaces the sampling gap artifact by a frequency smoothing artifact, it will be shown that the overall effect is conservative.

The paper is organized as follows: Section 2 describes the basic numerical methods testing methodology. Section 3 reviews the classical windowing method for the SA found in the literature and presents the proposed approach. The results are reported in Section 4 and discussed in Section 5. Finally, the conclusions are drawn in Section 6.

2. NUMERICAL METHODS

This section describes the conditions and results of the numerical experiments done by Monte Carlo simulations.

2.1. Nonlinear Method

The nonlinear method used in this paper is the FD calculated by the Higuchi's method [15]. A similar use of FD in the context of SA was notably used in [16] for magnetoencephalography (MEG) signals. The FD is easily calculated, even with a low number of data points. The method necessitates the selection of the time intervals (k). For speed, we only used k from 1 to 5.

2.2. Window method

In [12], different windows were tested. In this paper different systems are tested. Hence, the best window proposed in [12], the Welch Window [17], is used throughout this paper. The Welch Window is a parabola centered at N/2, as shown in Fig. 1 (b):

$$w(n) = 1 - \left(\frac{2n - (N+1)}{N-1}\right)^2 \quad (1)$$

2.3. Test Signal

The AR(1) process is used for the tests. It follows the relation:

$$x(n) = \alpha_1 x(n-1) + e(n) \quad (2)$$

where x is the signal, α_1 is the process parameter controlling the cut-off frequency and e is an independent identically distributed random variable called process noise. In this paper, the noise has a normal distribution. This process was used in [12] with $\alpha_1 = 0.995$. This choice was appropriate in order to highlight the windowing effect. The sampling gap in FFT is much more obvious in low pass signals. We used the same α_1 in most of this paper analysis. However, we also show results with α_1 =0.9, which clearly reduce the sampling gap problem and give an advantage to the System 1, with no windowing. An AR(1) process with $\alpha_1 = 0.9$ is shown in Fig. 2 (d). As in [12], the first data were removed in order to attain the steady state regime. The transient regime can be seen as non-stationary, which can affect the SA. We used an extra 2000 data points in order to have a relative impact of 5×10^{-5} in the case where $\alpha_1 = 0.995$.

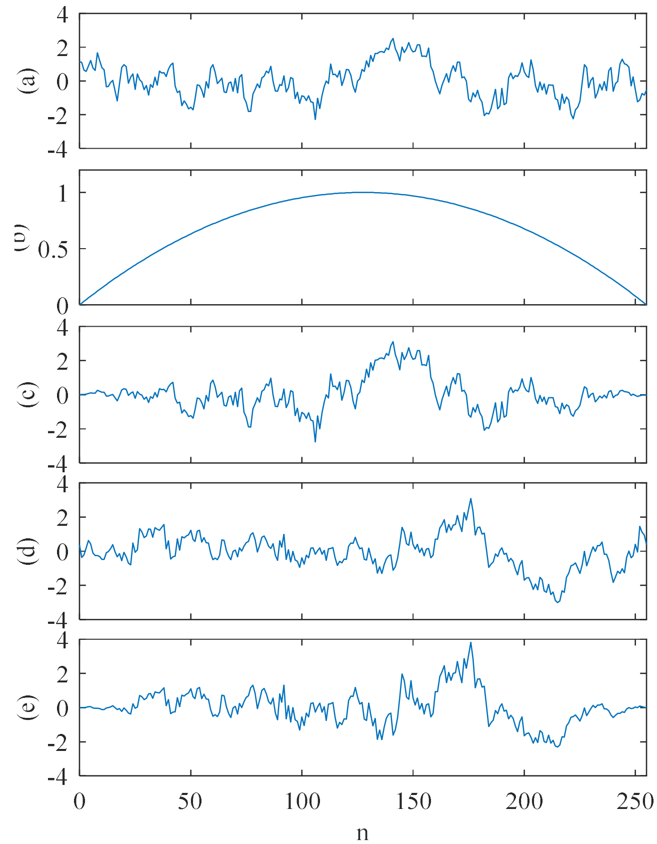


Fig. 1. Time domain representation of an example of 256 data points of an AR(1) signal with $\alpha_1 = 0.9$ and its processed versions by the proposed system. The signal variables are given with respect to the proposed system shown in Fig. 2 (d). The figure presents the original signal x (a), the Welch window (b), the original windowed signal x_w (c), a surrogate of the windowed signal s (d) and the windowed surrogate signal s_w (e).

2.4. Performance Measure: The False Rejection Rate

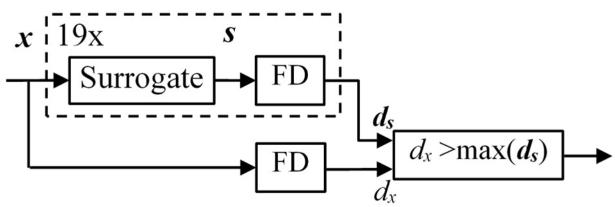
The FRR is used to compare the performances of the different systems. It is often called "Type 1 error." A perfectly fair or balanced test should give a FRR of exactly 5%. When the FRR is lower, the test is considered conservative while if it is higher, the test is said to be optimistic.

2.5. Monte Carlo Simulations

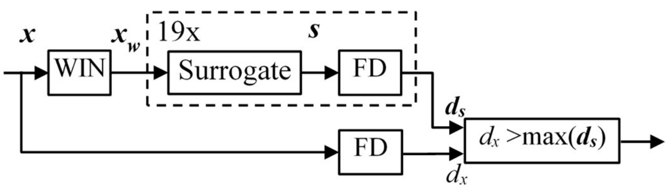
Monte Carlo simulations were conducted in order to obtain the FRR of the different systems. The number of tests per point in these graphs was 20 000. The same signals were tested for every method. This gives a maximum error of 1% at 3 standard deviations when the FRR is of 50% and 0.2% when FRR is at 5%. In the first simulation, α_1 was set to 0.995. These are shown in Fig. 3 (a-c). The right-sided (a), left-sided (b) and bilateral tests results are shown. In (d), the results for the bilateral test is given for $\alpha_1 = 0.9$.

3. SURROGATE ANALYSIS AND WINDOWING

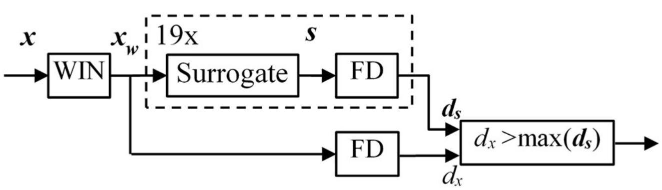
The SA compares a nonlinear feature, such as the Fractal Dimension (FD) [15] used in this paper, of a signal to the distribution same feature obtained on random signals with identical Power Spectrum. To do this, surrogate signals respecting the null hypothesis with the same power spectrum as the originals signals must be generated. The most common approach, which is considered here, is the phase randomization in the frequency domain. First, the Fast Fourier Transform (FFT) is applied. The phase is set to a uniform distribution between 0 and 2π, with conjugate symmetry in order to preserve real values. Then, the signal is set back in the time domain by inverse Fourier transform (IFFT).



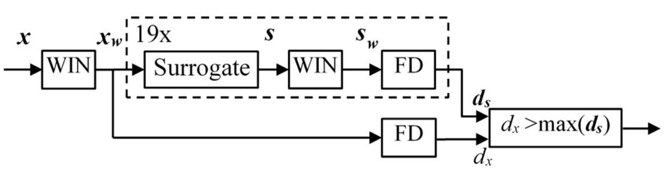
System 1: No Windowing
(a)



System 2: Windowing Applied as in [12]
(b)



System 3: Same Spectrum
(c)



System 4: Proposed System
(d)

Fig. 2. Windowing in the surrogate analysis system for right-sided tests. The system with no windowing is presented in (a). The method used in [12] is shown in (b), with the only difference being that the FD is used instead of Local Linear Prediction. The system in (c) is a variation of the previous one in which the FD of the original series is obtained from the windowed original series.

For a unilateral test with an aimed 5% FRR, 19 such surrogate series are generated. The FD is calculated for every surrogate series. The final stage is a comparison between the FD of the original series to those of the surrogate. A positive result to the test is given if the original series FD is higher than the surrogate series FD when the test is right sided and lower when the test is left sided. For a bilateral test, the number of surrogate series must be increased at 39 and the test will be positive if the original FD is higher of lower than all the surrogate series FD.

The null hypothesis is that the signal can be produced by a stationary linearly filtered white Gaussian noise. If the data processing produces some artifacts that are not of an ARMA type, these may influence the results.

When windowing is applied on a signal, two effects must be considered:

1) The spreading of the power spectrum (explained by the convolution theorem)
2) A non-stationarity in the variance of the signal.

These two effects can affect the nonlinear feature of the signal. It is important to note that non-stationarity is removed by phase randomization. If some windowing artifact are present in the original signal but not in the surrogate (or the reverse), a bias is created.

The four systems presented in Fig. 2 show different approaches for applying windowing in SA. The systems are shown for right-sided tests with FRR set at 0.05 (if the test was unbiased). The rest of the present sections describes the first three systems used as a basis of comparison and the proposed system.

3.1. System 1: Windowless Surrogate Analysis

The basic SA without windowing is presented in Fig. 2 (a). It is the system mostly used throughout the literature proposed in [1]. It can be interpreted as using a rectangular window. Here, the main differences between the original signal and its surrogate are the sampling gap and correlation between the frequency bins phases [1]. Although there is a very high spreading of the Power Spectrum, it is identical between the original and the surrogate signals.

3.2. System 2: Method of Suzuki [12]

In Fig. 2 (b), the windowing in applied before the FFT. However, the FD of the original series is calculated on the *unwindowed* version. Although the sampling gap effect is strongly reduced, the Power Spectrum differs between the original and surrogate series. Moreover, this difference is identical throughout the surrogate series. On the other hand, all the signals are stationary. The method was possibly first briefly mentioned in the original SA paper [1] but was really analyzed in [12].

3.3. System 3: Same spectrum

A final reference method is given in Fig. 2 (c). The system is similar to the method in (b), but the FD of the original series is calculated on the *windowed* version. Therefore, the original and surrogate series have the same Power Spectrum.

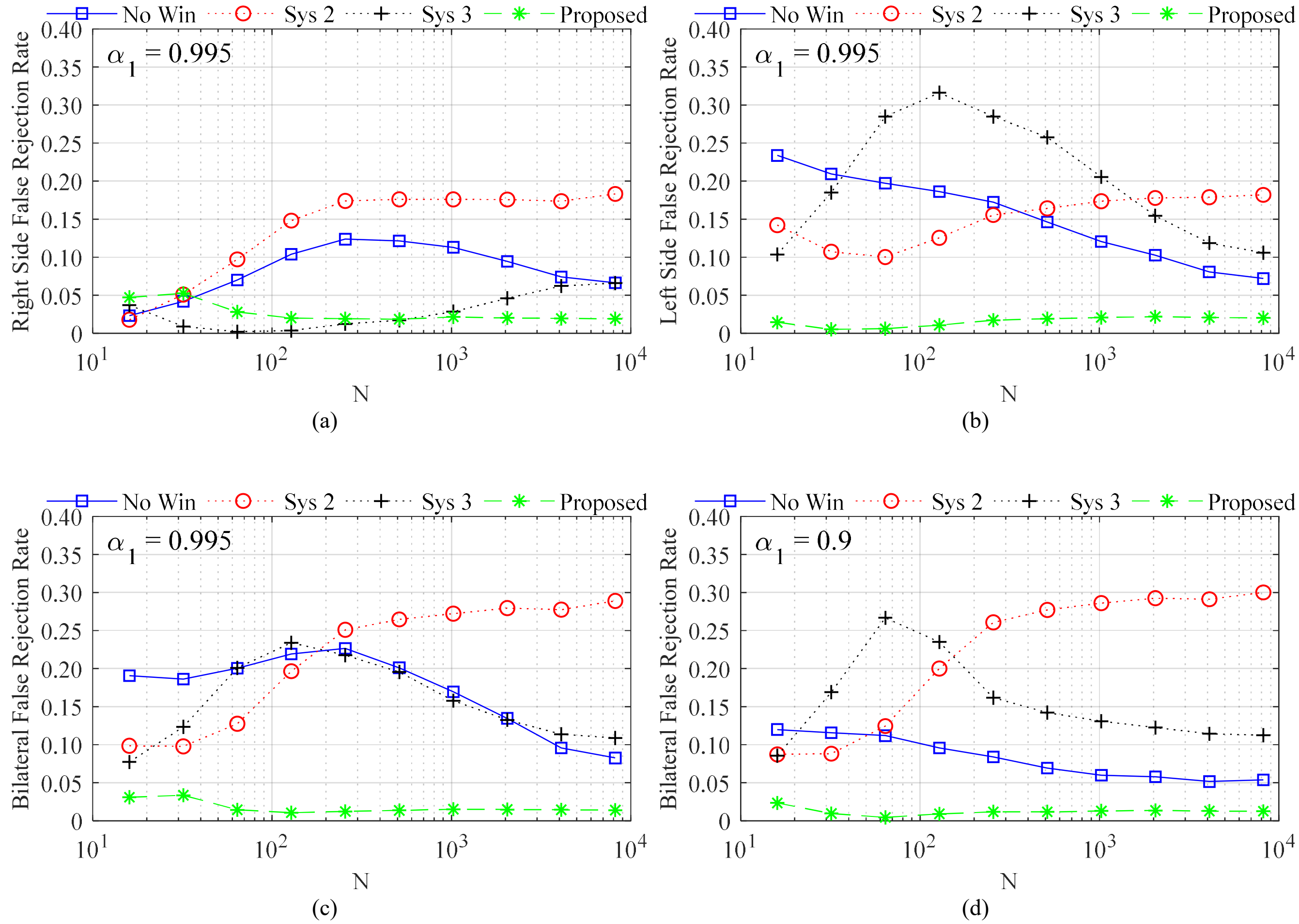


Fig. 3. False Rejection Rates vs the number of data points (window size) of all systems for an AR(1) process with α_1 = 0.995 for (a-c). Right-sided (a), left-sided (b) and bilateral (c) tests are shown. In (d), the process has α_1 = 0.9 and the test is bilateral. The number of tests was 20 000 per points.

However, while the surrogate series are stationary, the windowed original signal is not. This method has not been found clearly explained in the literature. In [1], the idea of using a windowing method was proposed. The exact application was, however, not concretely described, but it could be assumed that it was that of System 2 or System 3. Although System 3 is an obvious variant with the System 2, it is included in the numerical tests for the sake of completion, because it shows a situation where the only artifact is the non-stationarity.

3.4. System 4: Proposed Windowing Method for Surrogate Analysis

The proposed system is reported in Fig. 2 (d). The difference with (c) is that a second windowing stage is added to the surrogate series before calculating the FD. The two main relations between the original and surrogate series are:

1) The non-stationarities are similar
2) The Power Spectrums are different between the surrogates.

The first aspect ensures that the bias in the nonlinear method caused by the window are similar. The second aspect adds variability in the surrogate series. The Power Spectrums of the surrogate will be approximately centered to the windowed original signal's Power Spectrum. For this reason, it is expected that the test will be more conservative. Examples of the different signals obtained in the System 4 are shown in Fig. 1 (a) (c-e) along with the window used (b). The windowed signal is non-stationary.

4. RESULTS

A bias in the FD of the surrogate will reduce the FRR of one of the one-sided tests while increasing the FRR of the other. For example, the System 3 shows a very low FRR for the right-sided test in Fig. 3 (a) but very high for the left-sided test. The effect of the bias for bilateral test is harder to predict. When a normal distribution is considered, the bias simply increases the FRR. However, when the distribution is skewed, the FRR can be either lowered or raised (or in some particular case stayed unaffected.) Adding some Kurtosis effects, it becomes necessary to simply rely on simulations to assess the impact of the different biases. In Fig. 3 (a-c), the System 2 bilateral test has some FRRs higher than for both one-sided tests while the system 3 does not. The relation between the standard deviation of the offsets and standard deviation of the surrogates FDs will impact both one-sided tests, as well as the bilateral test.

The results show that the proposed system is generally very conservative. Since the right-sided tests FRR is higher than for the left-sided tests, there is still a weak bias. The bilateral test FRR is almost always at 1%. The proposed system is the only one which can give conservative results in bilateral tests.

When α_1 = 0.9 (d), the impact of not using windowing (System 1) is much weaker, as expected. Also, as the number of data points increase, the windowing problem disappear. The number of data points necessary to remove the need of windowing depends on the bandpass of the signal, controlled by α1 in this case. The System 2 has the same kind of behaviour with FD that it had with Local Linear Prediction as used in [12]. It has better performances than System 1 (no window) when the number of data points is small, but worse when the number is higher. Although the System 3 performs better than System 2 when the number of data points is high, it almost never performs better than the System 1 and 2 simultaneously.

5. DISCUSSION

The results shown in this paper were based on a certain nonlinear method (FD), a particular window (Welch) and a specific type of signal, the AR(1) process with very low pass characteristics. It must be emphasized that every time the SA is used, a careful examination of the null hypothesis should be carried out with the selected nonlinear method and window on signals with similar power spectrum as the data. In other words, the analysis carried out in this paper should be done for any new combination of nonlinear function, window and signal. The SA should never be used blindly [8]. As it should be reminded in every paper about the SA, the interpretation of the results must be limited to rejecting or not that “a linear, Gaussian, stationary, stochastic dynamical process underlies the data” [18].

In this paper, we used the Welch’s window because it was the recommended window in [12]. However, this choice may be improved for the proposed system. Obviously, the optimal window depends strongly on the nonlinear method used and the type of signal analysed.

Finally, the SA has variants in which the null hypothesis includes a static (memoryless) monotonic nonlinear transform such as Amplitude Adjusted Fourier Transform (AAFT) [1] or Iterated AAFT (IAAFT) [19]. The proposition of this paper can be extended to this variant with the same expected benefits.

6. CONCLUSION

The aim of this paper was not to show a ready-to-use method to detect nonlinearity. Rather, it proposed a system that must be adapted and tested every time nonlinear analysis is used when the number of data points is limited. The method proposed, although very conservative, allows to rule out the effect of windowing of the already complicated interpretation of the surrogate analysis.

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REFERENCES

[1] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, and J. D. Farmer, "Testing for nonlinearity in time series: the method of surrogate data," *Physica D: Nonlinear Phenomena,* vol. 58, no. 1-4, pp. 77-94, 1992.

[2] E. Bradley and H. Kantz, "Nonlinear time-series analysis revisited," *Chaos: An Interdisciplinary Journal of Nonlinear Science,* vol. 25, no. 9, p. 097610, 2015.

[3] C. J. Stam, "Nonlinear dynamical analysis of EEG and MEG: review of an emerging field," *Clinical neurophysiology,* vol. 116, no. 10, pp. 2266-2301, 2005.

[4] R. G. Andrzejak, K. Lehnertz, F. Mormann, C. Rieke, P. David, and C. E. Elger, "Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: Dependence on recording region and brain state," *Physical Review E,* vol. 64, no. 6, p. 061907, 2001.

[5] M. Caza-Szoka, D. Massicotte, F. Nougarou, and M. Descarreaux, "Surrogate analysis of fractal dimensions from SEMG sensor array as a predictor of chronic low back pain," in *2016 38th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, 2016: IEEE, pp. 6409-6412.

[6] M. Caza-Szoka and D. Massicotte, "Detection of Non Random Phase Signal in Additive Noise with Surrogate Analysis," in *ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2019: IEEE, pp. 1159-1163.

[7] L. Faes, H. Zhao, K. H. Chon, and G. Nollo, "Time-varying surrogate data to assess nonlinearity in nonstationary time series: application to heart rate variability," *IEEE transactions on biomedical engineering,* vol. 56, no. 3, pp. 685-695, 2008.

[8] D. Kugiumtzis, "Test your surrogate data before you test for nonlinearity," *Physical Review E,* vol. 60, no. 3, p. 2808, 1999.

[9] T. Schreiber and A. Schmitz, "Surrogate time series," *Physica D: Nonlinear Phenomena,* vol. 142, no. 3-4, pp. 346-382, 2000.

[10] F. J. Harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proceedings of the IEEE,* vol. 66, no. 1, pp. 51-83, 1978.

[11] J. Theiler, "Spurious dimension from correlation algorithms applied to limited time-series data," *Physical review A,* vol. 34, no. 3, p. 2427, 1986.

[12] T. Suzuki, T. Ikeguchi, and M. Suzuki, "Effects of data windows on the methods of surrogate data," *Physical Review E,* vol. 71, no. 5, p. 056708, 2005.

[13] J. Timmer, "Power of surrogate data testing with respect to nonstationarity," *Physical Review E,* vol. 58, no. 4, p. 5153, 1998.

[14] R. G. Andrzejak, K. Schindler, and C. Rummel, "Nonrandomness, nonlinear dependence, and nonstationarity of electroencephalographic recordings from epilepsy patients," *Physical Review E,* vol. 86, no. 4, p. 046206, 2012.
[15] T. Higuchi, "Approach to an irregular time series on the basis of the fractal theory," *Physica D: Nonlinear Phenomena,* vol. 31, no. 2, pp. 277-283, 1988.
[16] C. Gómez, Á. Mediavilla, R. Hornero, D. Abásolo, and A. Fernández, "Use of the Higuchi's fractal dimension for the analysis of MEG recordings from Alzheimer's disease patients," *Medical engineering & physics,* vol. 31, no. 3, pp. 306-313, 2009.
[17] P. Welch, "The use of fast Fourier transform for the estimation of power spectra: a method based on time averaging over short, modified periodograms," *IEEE Transactions on audio and electroacoustics,* vol. 15, no. 2, pp. 70-73, 1967.
[18] J. Timmer, "What can be inferred from surrogate data testing?," *Physical Review Letters,* vol. 85, no. 12, p. 2647, 2000.
[19] T. Schreiber and A. Schmitz, "Improved surrogate data for nonlinearity tests," *Physical review letters,* vol. 77, no. 4, p. 635, 1996.